

Collaborative Localization of Vehicle Formations Based on Ranges and Bearings

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Abstract—We examine the problem of jointly determining the positions of multiple underwater vehicles based on a set of pairwise range and bearing measurements taken over time. This extends prior work on the so-called (static) collaborative localization paradigm where a hybrid approach was proposed for seamless instantaneous fusion (i.e., no time dependence) of range and bearing measurements. To incorporate time we add to the original convexified least-squares cost function a regularizing term that penalizes deviations between predicted and computed vehicle positions at a given instant. The method operates progressively over time, with past estimates used for prediction at the current instant assuming a very simple quasi-linear motion model. The method is amenable to parallelization, with simple gradient-like updates. Numerical results demonstrate promising accuracy gains (reduction on the order of 10 % in terms of root-mean-square positioning error) in simulations inspired by an underwater geoaoustic surveying application.

I. INTRODUCTION

The development of networked systems of agents that can interact with the physical world and carry out complex tasks in various contexts is currently a major driver for research and technological development [1]. This trend is also seen in contemporary ocean applications [2], and motivates the present work, which falls under the scope of EU H2020 project WiMUST. The goal of this project is to develop advanced control, communication and signal processing tools to enable a team of marine robots, either on the surface or submerged, to jointly conduct geoaoustic surveys and eventually replace current systems where a single vessel tows very long and cumbersome arrays of streamers [3] (Fig. 1). In this type of survey a powerful towed source produces acoustic waves that penetrate the sea bottom, and its layered structure is inferred from the pattern of echoes in the reflected acoustic field observed at a set of hydrophones over an extended period and wide geographic area. Such surveys are routinely carried out to characterize the sea bottom prior to underwater construction, for monitoring the condition of pipelines and submerged structures, and for exploration/exploitation of offshore oil and gas fields.

Unlike conventional systems, the absence of long physical ties between the surface ship and the data acquisition devices makes it easy to adjust the spatial sampling configuration in

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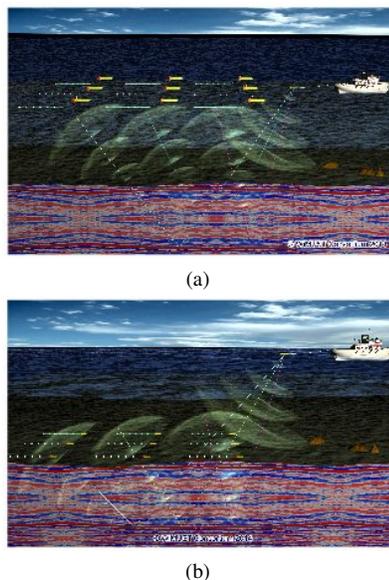


Fig. 1. Geoaoustic surveying in WiMUST using a team of autonomous vehicles to tow multiple streamer arrays (a) 2D surface configuration (b) 3D submerged configuration

WiMUST by changing the shape and trajectory of the vehicle formation. This flexibility provides potentially useful additional degrees of freedom for estimating the layered structure of the bottom.

Coordinated operation of vehicles requires the existence of a communication network to share data, most critically, those related to navigation and positioning [4]. This work is concerned with localization of (underwater) vehicles, a key subsystem needed in the absence of GPS to properly georeference any acquired data and also used in multi-vehicle cooperative control algorithms. In WiMUST, e.g., the acoustic signals acquired by streamers must be georeferenced to high precision to enable successful inference of deep sub-bottom layers. Shared positioning data may include measurements such as inter-vehicle distances or bearings, needed to compute spatial coordinates. Designing efficient sharing methods in low-rate (broadcast) acoustic channels¹ is practically important [6], [7], but here we take the existence of such physical-level mechanisms for granted, and assume that measurements are available as needed for vehicles to compute their positions.

¹The existence of alternative vehicle-to-vehicle data links operating at distances on the order of 10 m (e.g., optical [5]) is quite helpful in scenarios that are relevant to WiMUST.

The problem class of interest here is collaborative, or network, localization, where multiple vehicles jointly determine their positions by sharing pairwise measurements and measurements to a set of reference points (anchors). These measurements are typically ranges in GPS-like systems [6], while in recent work we have developed a hybrid fusion method based on convex relaxation that seamlessly integrates range and bearing measurements to attain more accurate and robust operation [8]. The algorithm itself is readily parallelizable, computationally very simple, and attains superior accuracy when compared with other formulations of collaborative optimization based on convex relaxation — their main advantage being the existence of a single (global) minimum that can usually be found numerically in a robust way. When vehicles are equipped with inertial sensors that allow them to determine the absolute bearing to any other vehicle within visual range, as in WiMUST, the algorithm becomes fully distributed in the classic sense: In an underlying connectivity graph nodes acquire local measurements and the algorithm proceeds through exchanges between neighbouring nodes. The presence of short-range inter-vehicle optical communication links [5] is useful not only for enabling fast data exchanges, but also as an additional means to determine relative bearings, due to the need for proper alignment in such links.

The collaborative localization algorithm of [8], termed CLORIS, operates on measurement snapshots, i.e., at a given instant in time pairwise ranges and bearings are collected (in practice, time skew between measurements are inevitable due to the limitations of ranging over a shared acoustic channel), and CLORIS is invoked to jointly determine the unknown positions. At the next snapshot the whole process starts anew, ignoring previous estimates.

In the present work we incorporate time into the estimation process, the goal being to further improve the accuracy of CLORIS by mitigating discrepancies between the estimated positions of individual vehicles over consecutive snapshots. We adopt a simple strategy from [9] that adds a regularizing term to the cost function used for collaborative localization in a given snapshot, penalizing deviations between estimated vehicle positions and those predicted based on previous snapshots. Prediction is very simple, based on a quasi-constant velocity model updated on a sliding time window. While the penalization and prediction strategy is inspired by [9], our relaxation approach for convexifying the problem is completely different, and therefore estimation algorithms are unrelated to those given in [9].

The quasi-constant velocity model is somewhat restrictive, but it provides significant gains in accuracy (about 10 % reduction in root mean-square error relative to localization with a single snapshot in our simulations) when individual vehicle trajectories are predominantly linear. While more sophisticated prediction methods could be devised, the linear motion model is well suited to WiMUST, as geoacoustic surveys usually require streamers to be towed along (parallel) linear trajectories most of the time.

Abstracting the processes through which range and bearing measurements are acquired and made available to localization algorithms [6], [7], we represent the vehicle formation at a given instant as an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, whose nodes are the sensors/vehicles and the arcs represent pairwise measurements. We also assume the existence of a set of reference points (anchors) deployed at fixed positions in the environment, relative to which measurements are taken, to address translation and rotation ambiguities in our formulation. Below, we will discuss manual and automatic calibration of anchor positions.

Let $x_i \in \mathbb{R}^n$, $i \in \mathcal{V}$ denote the position of node i at a given time ($n = 2$ or 3 for 2D or 3D localization, respectively), and similarly let $a_k \in \mathbb{R}^n$ be the position of anchor k . We define \mathcal{A}_i as the subset of anchors visible to node i , which is further split into those providing range (\mathcal{R}_i) or bearing (\mathcal{T}_i) measurements. The ranges between node i and node j or anchor k are given by $d_{ij} = \|x_i - x_j\|$ or $d_{ik} = \|x_i - a_k\|$. Similarly, bearings are encoded by unit-norm vectors, $u_{ij} = \frac{x_i - x_j}{\|x_i - x_j\|}$ or $u_{ik} = \frac{x_i - a_k}{\|x_i - a_k\|}$, depending on the measurement being relative to node j or anchor k . Actual measurements are corrupted by noise.

The collaborative hybrid localization problem consists in estimating, for each time instant t , the node positions $x_i \in \mathcal{V}$ from the available measurements d_{ij}, d_{ik} and u_{ij}, u_{ik} . In previous work [10] we have proposed a hybrid method for single-source localization seamlessly fusing ranges and bearings (FLORIS). Later, we have extended it to the collaborative paradigm [8] (CLORIS). We adopt a least-squares (LS) formulation motivated by the form of the likelihood function for range-based localization under Gaussian noise. The key feature of our approach is to replace nonlinear/non-convex LS range-based terms of the form $(\|\Delta x\| - d)^2$ by the convex underestimator $D_{B_d}^2(\Delta x) = \begin{cases} (\|\Delta x\| - d)^2, & \text{if } \|\Delta x\| \geq d \\ 0, & \text{otherwise.} \end{cases}$

The notation $D_{B_d}^2(\Delta x)$ emphasizes that this is the squared distance of the difference of coordinates Δx to B_d , a ball of radius d centered at the origin. This so-called *disk relaxation* is very precise and leads to a simple, fast and parallelizable range-based localization algorithm using gradient descent [11].

For hybrid localization, we add to the cost function new quadratic terms that quantify the squared distance from Δx to lines L going through the origin. For bearing u we have $D_{L_u}^2(\Delta x) = \|(\mathbf{I} - uu^T)\Delta x\|^2$ [10]. The hybrid cost function of CLORIS for static cooperative localization is thus [8]

$$\begin{aligned} f(\mathbf{x}(t)) = & \sum_{i \sim j} D_{B_{ij}}^2(x_i - x_j) + \sum_i \sum_{k \in \mathcal{R}_i} D_{B_{ik}}^2(x_i - a_k) \\ & + \sum_{i \sim j} D_{L_{ij}}^2(x_i - x_j) + \sum_i \sum_{k \in \mathcal{T}_i} D_{L_{ik}}^2(x_i - a_k), \end{aligned} \quad (1)$$

in which \mathbf{x} is the vector of concatenated node coordinates and $i \sim j$ denotes the existence of measurements involving nodes

i and j , more specifically of range measurements, $i \overset{\mathcal{R}}{\sim} j$, or angle measurements $i \overset{\mathcal{T}}{\sim} j$.

Therefore, on the one hand, (1) penalizes deviations of the node position estimates x_i from balls with radii d_{ij} , d_{ik} centered at other nodes or anchors, and, on the other hand, penalizes deviations of x_i from lines with orientation u_{ij} , u_{ik} going through other nodes or anchors.

III. LOCALIZATION OF MOVING VEHICLE FORMATIONS

In dynamic settings, anchors maintain their positions², while sensors move freely over time according to the model

$$\mathbf{x}(t) = \mathbf{x}(t-1) + \mathbf{v}(t)\Delta T, \quad (2)$$

where, similarly to $\mathbf{x}(t)$, vector $\mathbf{v}(t)$ stacks the velocity vectors for all nodes at (discrete) time $t \in \mathbb{Z}$, and ΔT is the time difference (in seconds) between snapshots.

We address the dynamic collaborative localization problem by estimating $\mathbf{x}(t)$ for each t in much the same way as in the static case, but using knowledge of previous estimates to form a prior on expected positions. Specifically, we follow the approach of [9] and modify the cost function (1) to obtain the optimization problem

$$\underset{\mathbf{x}(t)}{\text{minimize}} f(\mathbf{x}(t)) + \lambda \|\mathbf{x}(t) - \hat{\mathbf{x}}(t)\|^2, \quad (3)$$

where $\hat{\mathbf{x}}(t)$ denotes the prediction of $\mathbf{x}(t)$ based on previous snapshots and parameter $\lambda \in \mathbb{R}^+$ controls the amount of regularization. The prediction $\hat{\mathbf{x}}(t)$ uses (2), but with velocities estimated from coordinate differences in a number of previous snapshots. Filtering to obtain estimated velocities, $\hat{\mathbf{v}}(t)$, could be done in a number of ways, but here we follow the polar representation of [9] and compute magnitudes and phases through Weighted Moving Averages (WMA) as

$$\|\hat{v}_i(t)\| = \frac{\sum_{j=1}^{t-1} w_{\|v\|,j} \|x_i(t-j) - x_i(t-j-1)\|}{\Delta T \sum_{j=1}^{t-1} w_{\|v\|,j}}, \quad (4)$$

$$\angle \hat{v}_i(t) = \frac{\sum_{j=1}^{t-1} w_{\angle v,j} \angle(x_i(t-j) - x_i(t-j-1))}{\Delta T \sum_{j=1}^{t-1} w_{\angle v,j}}, \quad (5)$$

where $w_{\|v\|,j}$, $w_{\angle v,j}$ are weights that emphasize recent contributions, as they decay to zero for older position estimates. The weights used for magnitudes and phases need not be the same, and may be set to attain suitable tracking for various node trajectories. Still, we keep these weights static for the ensemble of our simulations.

Prediction by averaging is most effective if the terms in the sums (4), (5) are approximately constant, i.e., individual velocities are approximately constant (trajectories are approximately linear), but not necessarily equal across nodes. Our simulations confirm that best performance is indeed attained for linear trajectories, even if the magnitudes of velocity vectors vary moderately over time. By contrast, regularization in (3) becomes significantly less effective when, e.g., node trajectories are circular, even for constant velocity magnitudes.

²This simplification is not actually required. In our formulation anchors can move, as long as their position/attitude remains known over time.

Regarding solution algorithms for problem (3), note that the regularizer is separable and therefore introduces no coupling between different node coordinates. Then, the gradient of the modified cost function retains the same structure found for CLORIS, which enables it to be efficiently computed in parallel for each node using only information obtainable from neighbouring nodes. This property may be relevant to obtain a truly distributed collaborative localization solution involving physical vehicles, or simply as a convenient computational device for efficiently solving the optimization problem (3) at a central location. Development of tailored solution algorithms is beyond the scope of this paper, so in our simulations we use a general-purpose convex solver.

IV. SIMULATION RESULTS

We characterize the performance of the newly proposed method benchmarking it against CLORIS [8] in simulation. CLORIS minimizes (1) for each time instant t , whereas the proposed method solves (3) with velocities and node positions predicted according to (2), (4), (5).

We perform our tests in pre-defined networks, rather than on randomly generated ones, due to the fact that random graphs arising from pairwise measurements in collaborative scenarios often lead to ill-posed localization problems, in which a unique solution compatible with the data cannot be found [12].

In order to generate both range and bearing measurements we first add white Gaussian noise $w \sim \mathcal{N}(0, \eta^2 \|\delta_0\|^2 \mathbf{I})$, where η denotes the noise factor, to the nominal difference of node-node or node-anchor position vectors, $\delta_0 = x_i - x_j$ or $\delta_0 = x_i - a_k$, respectively, yielding $\delta = \delta_0 + w$. The noisy range and bearing measurements are generated according to $d = \|\delta\|$ and $u = \delta/\|\delta\|$.

We evaluate the estimation accuracy by the Root Mean-Square Error (RMSE) for the total trajectory followed by the vehicle formation. For a formation of N vehicles, moving during T time instants, and for a set of MC Monte Carlo runs, we define the RMSE as:

$$RMSE = \sqrt{\frac{1}{MC} \frac{1}{N} \frac{1}{T} \sum_{k=1}^{MC} \sum_{i=1}^N \sum_{t=1}^T \|x_i(t) - \tilde{x}_i(t)^k\|^2}, \quad (6)$$

where $x_i(t)$ and $\tilde{x}_i(t)^k$ respectively denote the true and estimated positions of the i -th node, for the t -th instant in the k -th Monte Carlo run. We ran sets of $MC = 100$ Monte Carlo trials for each measurement noise factor $\eta \in (0.001, 0.005, 0.01, 0.05, 0.1, 0.2)$, for 2D networks comprising 4 anchors, placed at the corners of a unit square, and $N = 10$ nodes animated with linear motion during $T = 10$ instants of time.

The following experiments were run using MATLAB R2015a and the generic convex solver CVX.

Example 1: Fig. 2 shows a comparison of the RMSE of the new method taking advantage of the network dynamics, and CLORIS, for a network whose nodes move all with the same constant linear velocity (0.1, 0.1). An improvement of approximately 10% in the position estimation accuracy is achieved

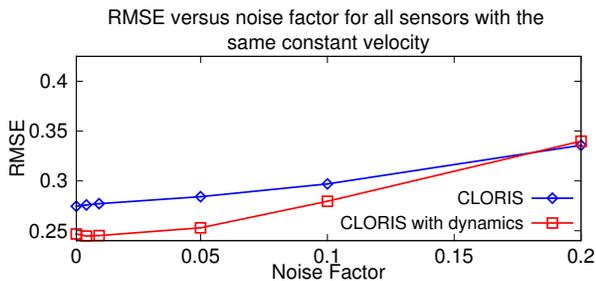


Fig. 2. Performance of the proposed method and of CLORIS: RMSE vs. noise factor for 4 anchors and 10 nodes moving at the same constant velocity during 10 time instants

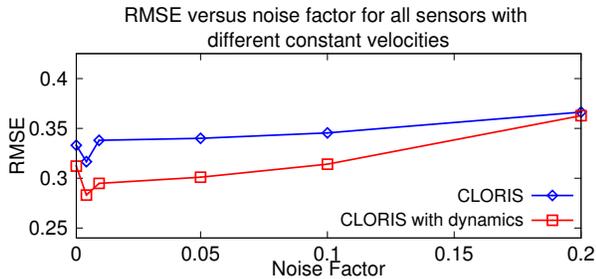


Fig. 3. Performance of the proposed method and of CLORIS: RMSE vs. noise factor for 4 anchors and 10 nodes moving at different but constant velocities during 10 time instants

for almost all noise factors by introducing the regularization term. Although this improvement seems to deteriorate when increasing the noise factor, for a considerable noise of 10% in the measurements, the new method still does better than the previous one.

Example 2: The performance of the two compared methods when the nodes move with different but *constant* rectilinear velocities, randomly generated in the interval $[0.05, 0.15]$, is depicted in Fig. 3. In this scenario the new proposed method also outperforms CLORIS in accuracy, for all noise factors.

Example 3: Fig. 4 depicts the performance results of both methods for a network in which nodes move according to different and *varying* linear velocities over time, in the interval $[0.05, 0.15]$. Again, regularizing with motion information leads to an improvement of about 10% over CLORIS, the gap being larger for lower noise factors. A scenario mixing the conditions of Examples 1 and 3, where a subset of nodes move with the same constant velocity while the remaining ones move with different and varying velocities over time, was also tested. The obtained accuracy results were consistent with the previous ones, with the proposed method outperforming CLORIS with a similar behaviour.

Example 4: In practical deployments it is sometimes inconvenient to assume that anchor positions are known. However, if anchors only provide range measurements it is possible to recover their geometric configuration up to a rotation and translation³ through factorization of the Euclidean Distance

³Anchors help to disambiguate flip ambiguities in EDM formulations [6], [13]. Their explicit inclusion in (1) also renders the proposed approach immune to such ambiguities in most cases.

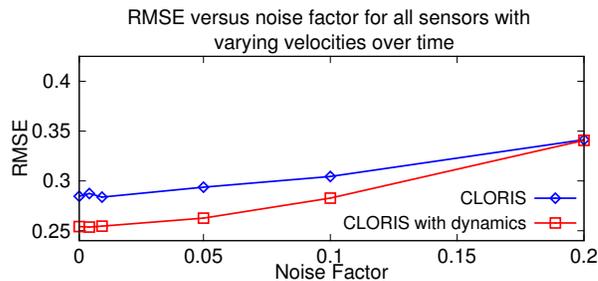


Fig. 4. Performance of the proposed method and of CLORIS: RMSE vs. noise factor for 4 anchors and 10 nodes moving at different and varying velocities, during 10 time instants

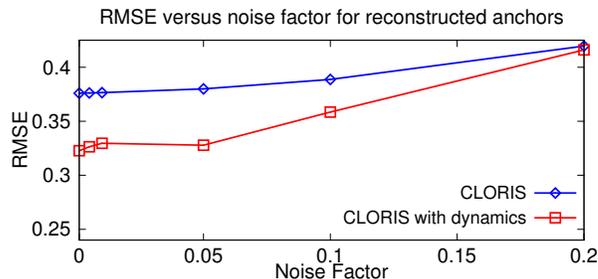


Fig. 5. Performance of the proposed method and of CLORIS: RMSE vs. noise factor for 4 reconstructed anchors through Euclidean Distance matrix factorization and 10 nodes moving at the same constant velocity, during 10 time instants

Matrix (EDM) generated by anchor-anchor ranges (see [13]). These computed anchor coordinates are then used as surrogates for the true ones in collaborative localization methods, enabling the determination of *relative* node positions and trajectories. Fig. 5 shows the accuracy results of estimating the node positions in a network under the same conditions of *Example 1* but with anchor coordinates reconstructed from the Euclidean distance matrix. In this scenario, the new proposed method introduces an improvement higher than 10% over CLORIS up to noise factor $\eta = 0.1$.

V. CONCLUSIONS AND FUTURE WORK

Our convex relaxation strategy for collaborative localization is markedly different from (range-based) semidefinite relaxation methods, as it allows us to retain node positions as optimization variables. This enables the inclusion of hybrid terms in the cost function, as well as regularization terms that induce temporal filtering of node coordinates, smoothing the estimated trajectories and reducing the RMSE by about 10% relative to instantaneous localization over several simulated scenarios with linear node velocities. Several improvements can be envisaged, such as: Developing better velocity prediction models to obtain similar performance gains over richer trajectories; deriving efficient parallel minimization algorithms for the regularized hybrid cost function; solving for multiple snapshots in a single optimization for even better accuracy.

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