

## Abstract

We present a **hybrid** method for **single-source localization in wireless sensor networks**, fusing noisy **range measurements** with **angular information** extracted from video. We develop and test a hybrid localization algorithm which surpasses the limitations of previous fusing approaches found in the literature. The proposed method (**FLORIS**) is based on a **nonconvex least-squares joint formulation**, for which a **tight convex relaxation** is applied to obtain a **semidefinite program**.

## 1. Introduction

- The “where am I” problem is a key issue in technology.
- GPS is not available neither indoors nor in underwater environments; these are challenging scenarios due to strong multi-path propagation, diffraction and interference leading to poor localization accuracy.
- The key to overcome these issues lies on exploring hybrid schemes. We fuse distances and angular information to localize a target in a wireless sensor network.
- Attempts to fuse these two types of information have been presented, but impose severe limitations.

## 2. Problem Formulation: Hybrid Source Localization (FLORIS)

Let  $x \in \mathbb{R}^n$  be the target position to be estimated based on  $m$  known reference points (anchors)  $a_i \in \mathbb{R}^n$ ,  $i = 1, \dots, m$ .

- Anchors in set  $\mathcal{R}$  provide range measurements to the source:  $d_i = \|x - a_i\| + w_i$ ,  $w_i$  is a noise term.
- Anchors in set  $\mathcal{T}$  measure bearings  $u_i$ , a perturbed versor centered around the true direction  $\frac{x - a_i}{\|x - a_i\|}$ .

We propose estimating the source position by minimizing the joint least-squares cost function:

$$f(x) = \sum_{i \in \mathcal{R}} D^2(x, C_i) + \sum_{i \in \mathcal{T}} D^2(x, L_i), \quad (1)$$

with  $D^2(x, C_i) = (\|x - a_i\| - d_i)^2$  and  $D^2(x, L_i) = (x - a_i)^T (I_n - u_i u_i^T) (x - a_i)$ .

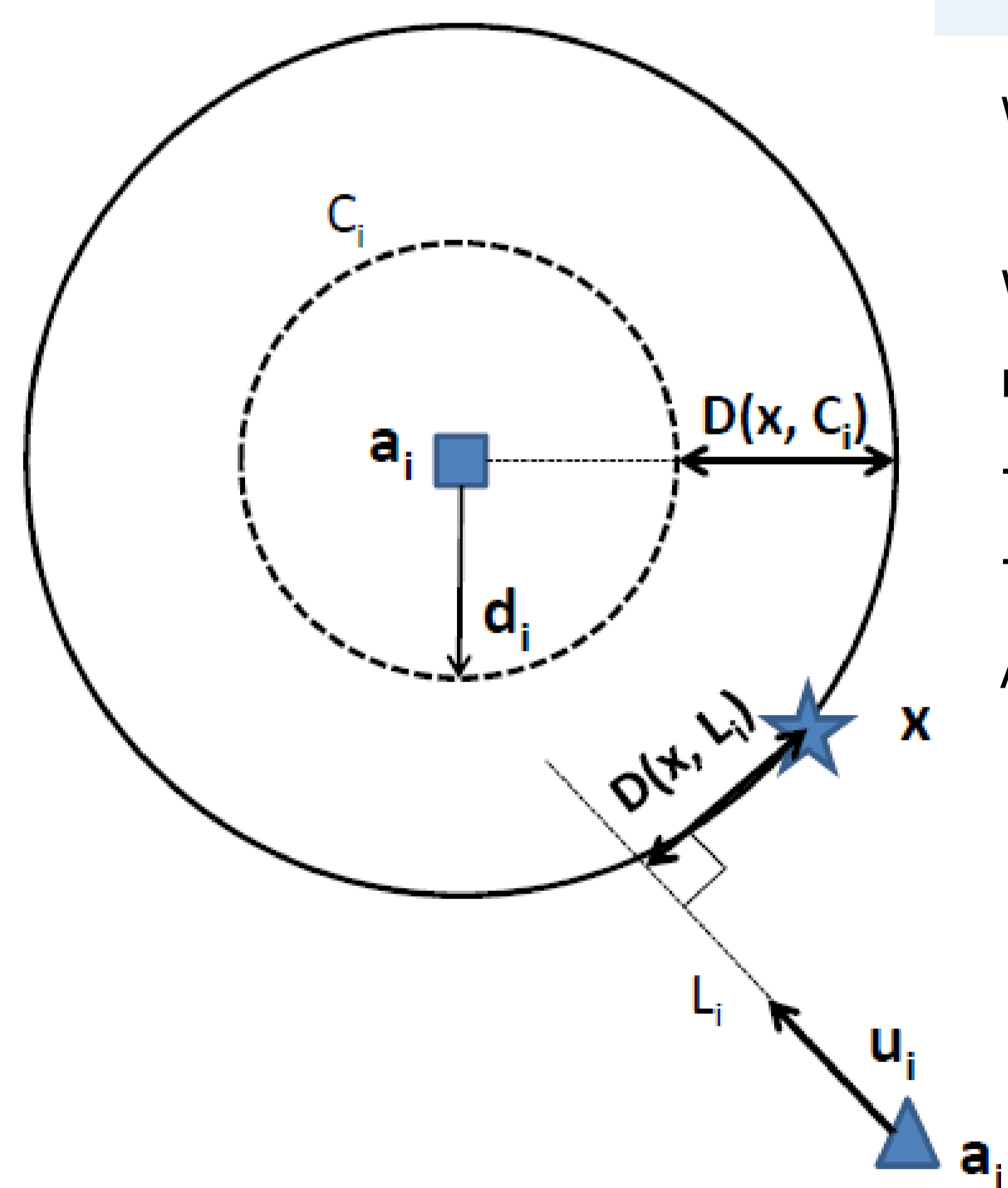
We attempt to balance the distances of the source position estimate relative to:

- the circles  $C_i$  centered at the wireless anchors with radii  $d_i$ ;
- the lines  $L_i$  originating at the visual anchors with orientation  $u_i$ .

An alternative parametrization for minimizing (1), inspired in [1] is:

$$\begin{aligned} & \underset{x, y_i, \theta_i, t_i}{\text{minimize}} \sum_{i=1}^m \|x - y_i\|^2 \\ & \text{subject to} \quad y_i = a_i + d_i \theta_i, \quad \|\theta_i\| = 1, \quad i \in \mathcal{R} \\ & \quad \quad \quad y_i = a_i + u_i t_i, \quad t_i \in \mathbb{R}^+, \quad i \in \mathcal{T} \end{aligned} \quad (2)$$

Given the constellation  $y_i$ ,  $x = \frac{1}{m} \sum_i y_i$  lies at the center of mass. Making  $y = a + R \begin{bmatrix} \theta \\ t \end{bmatrix}$ , problem (2) is written as:



**Fig. 1.** Geometric representation of terms in the joint cost function (1)

## FLORIS formulation:

Relaxed SDP

$$\begin{aligned} & \underset{W}{\text{minimize}} \quad \text{tr} \left( \begin{bmatrix} R^* J R & R^* J a \\ a^* J R & a^* J a \end{bmatrix} \begin{bmatrix} \theta \\ t \\ 1 \end{bmatrix} \begin{bmatrix} \theta^T & t^T & 1 \end{bmatrix} \right) \\ & \text{subject to} \quad W \geq 0 \\ & \quad \quad \quad \text{tr}(W_{i,i}) = 1, i \in \mathcal{R} \end{aligned} \quad (3)$$

$W_{i,nm+1} \geq 0, i \in \mathcal{T},$   
 $W_{nm+1,nm+1} = 1$   
 ~~$\text{rank}(W) = 1$~~

- Vectors  $\theta$  and  $t$  can be obtained by SVD factorization.
- The relaxation performed is tight (frequency of  $\text{rank}(W) = 1$  is high), hence the optimal solution is frequently found.

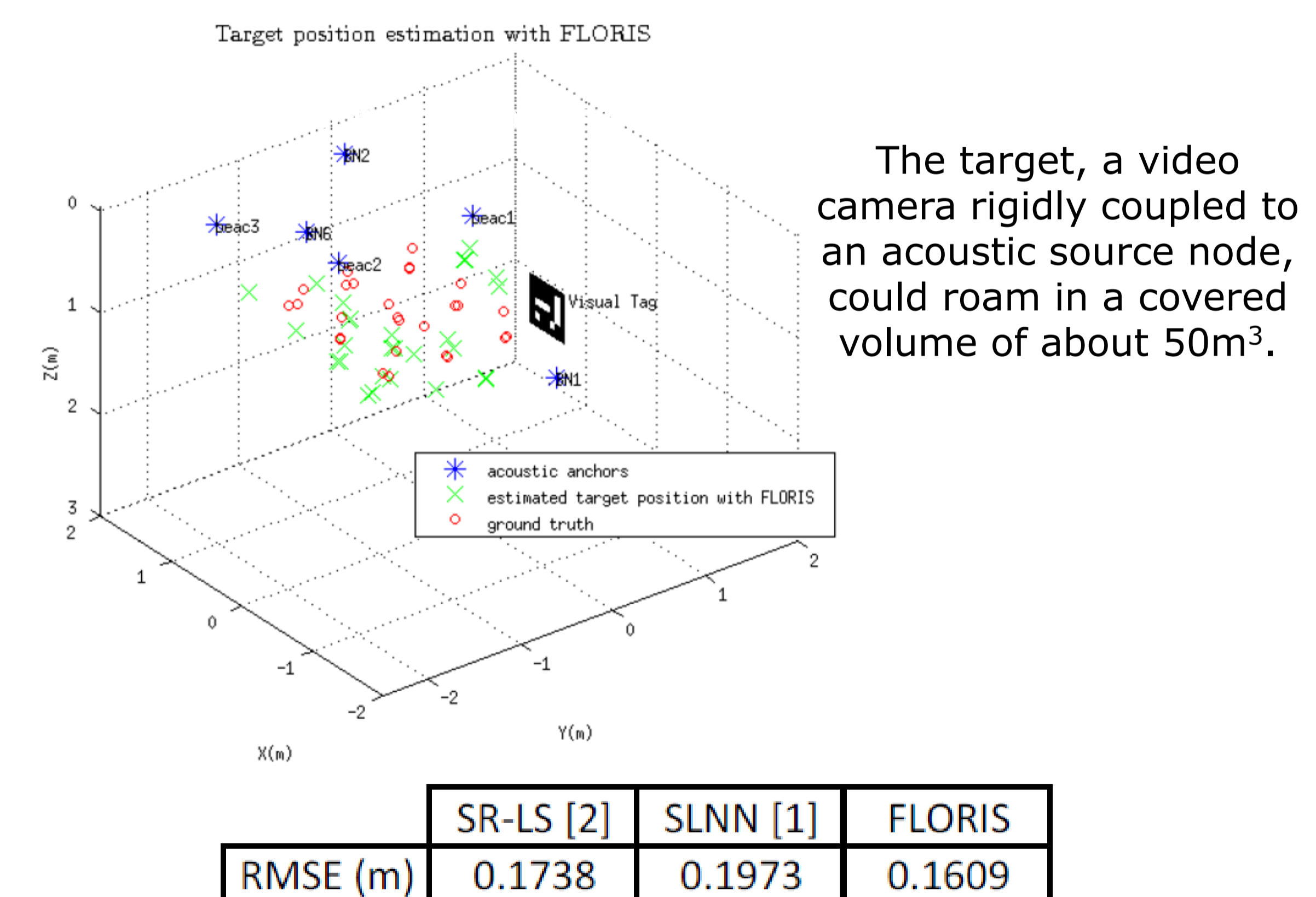
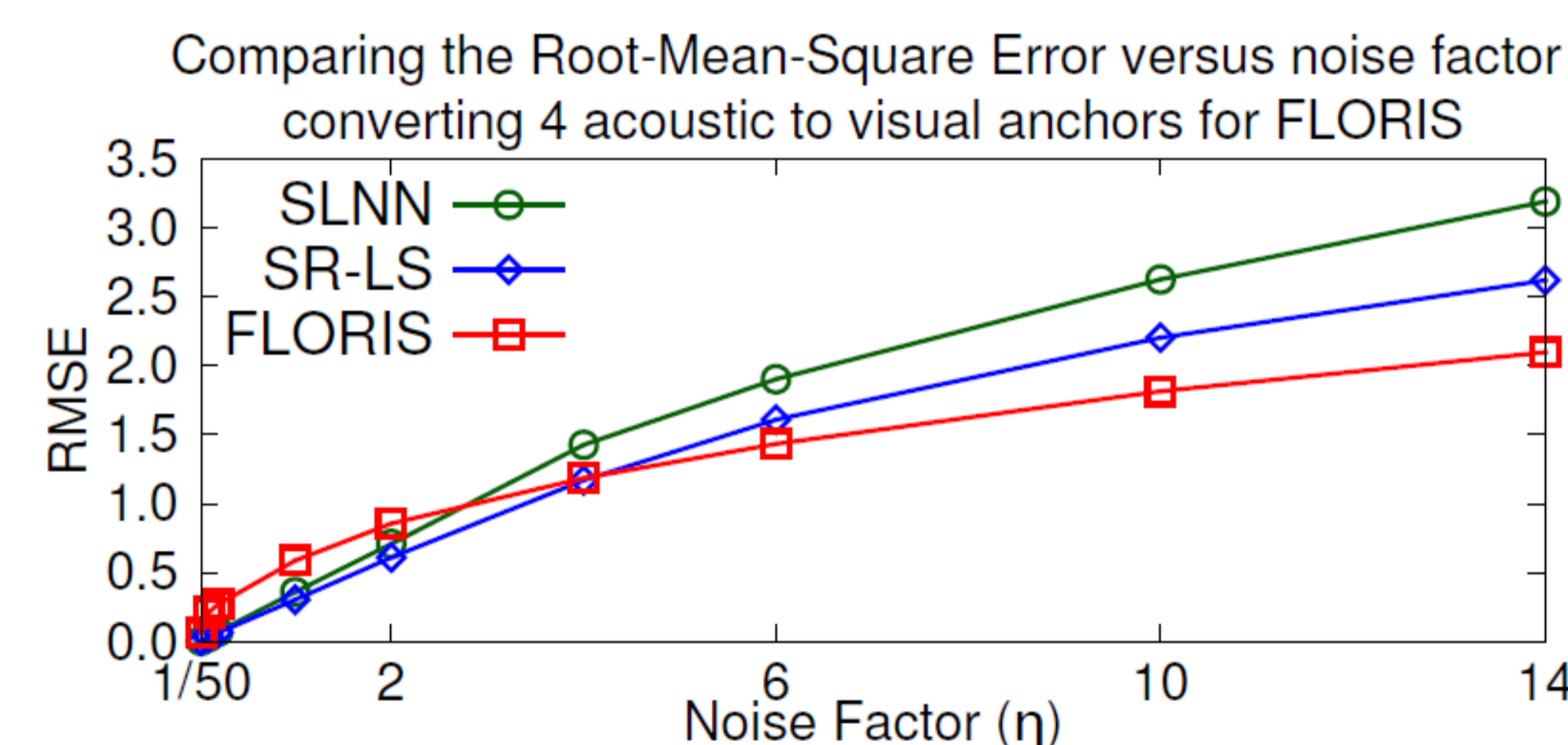
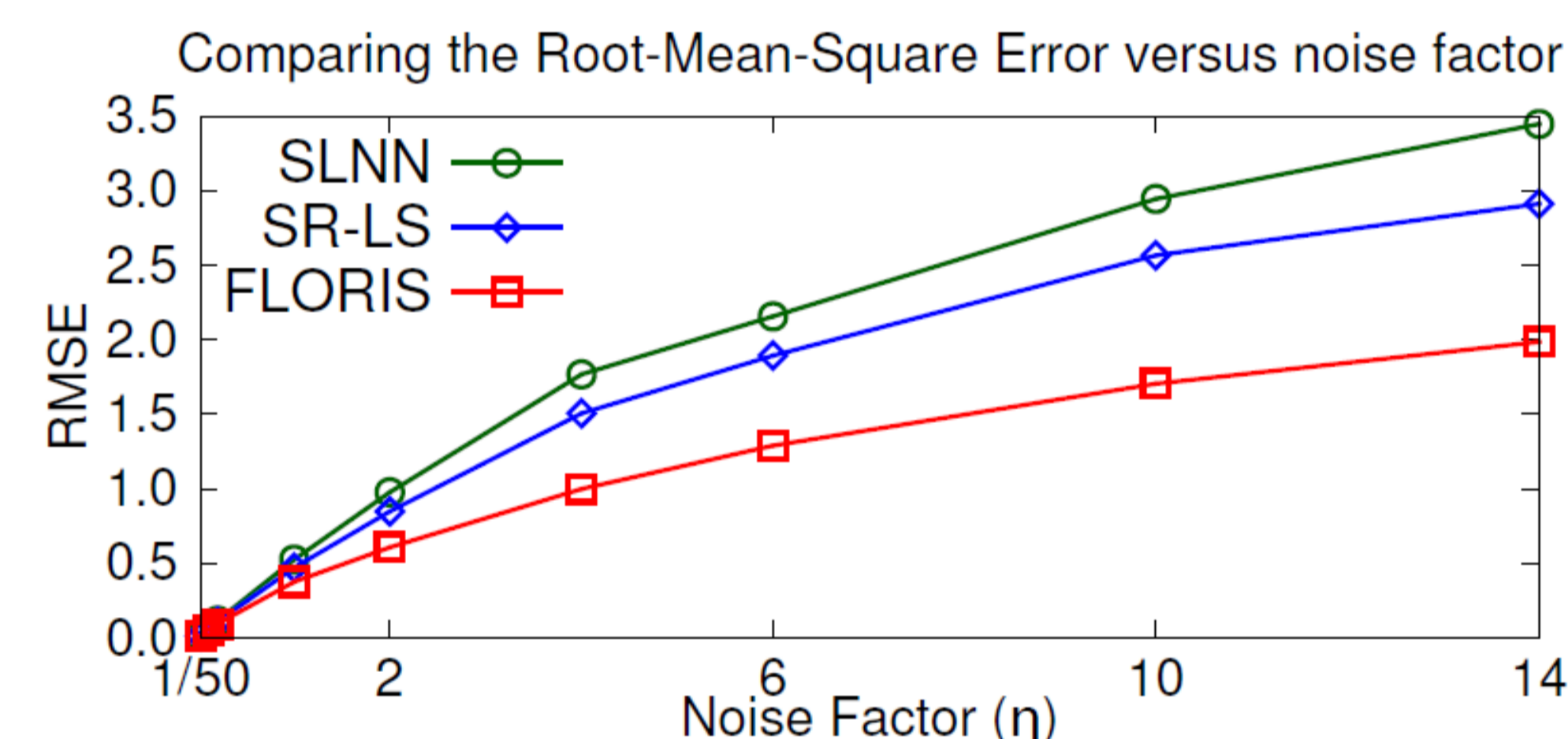
## 3. Results – 3D source localization performance

### Simulation results:

- Randomly generated networks of 6 acoustic and 4 visual anchors.
- Source randomly positioned.
- Noise factor ( $\eta$ ) is a scaling factor relative to an established baseline scenario reflecting the precision found in the experimental setup.
- 1000 Monte Carlo runs for each noise factor.

### Experimental results:

- Experimental set-up, consisting on acoustic anchors and augmented reality tags as visual anchors.



**Table 1.** 3D source localization performance comparison, for a data set obtained in the experimental set-up

## 4. Conclusions and future work

- FLORIS** is a **new** approach for centralized **localization** based on **nonlinear least squares** relying on a **tight SDP relaxation** that provides an **efficient solution** method using a generic convex solver.
- In numerical results FLORIS achieves **higher accuracy than state-of-the-art** methods, specially for **high noise** scenarios. In a **real indoor deployment** FLORIS shows **better accuracy** than the benchmarks.

## References

- [1] P. Oguz-Ekim, J. Gomes, J. Xavier, M. Stolic, and P. Oliveira, “An Angular Approach for Range-Based Approximate Maximum Likelihood Source Localization Through Convex Relaxation,” IEEE Transactions on Wireless Communications, vol. 13, no. 7, pp. 3951–3964, July 2014.
- [2] A. Beck and P. Stoica, “Exact and Approximate Solutions of Source Localization Problems,” IEEE Transactions on Signal Processing, vol. 56, no. 5, pp. 1770–1778, May 2008.

**Acknowledgments:** This research was partially supported by Fundação para a Ciência e a Tecnologia (FCT) through project FCT UID/EEA/50009/2013 and EU FP7 project MORPH (grant agreement no. 288704)