IST, Digital Signal Processing, Lab. #0 Report
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1. Sampling frequency $F_s = 10$KHz:
```
>> fs=1e4;
```
Vector with sampling time instants:
```
>> t=0:1/fs:2-1/fs;
```
Samples $x[n]$ of a 1Khz sine wave:
```
>> x=sin(2*pi*1e3*t);
```
$X[k]$, the DFT of $x[n]$:
```
>> xfreq=fft(x);
```
We plotted the magnitude of $X[k]$,
```
>> plot(0:length(xfreq)-1,abs(xfreq))
```
and observed that $|X[k]|$ is zero, except for $k = 2000$ and $k = 18000$.
The frequency corresponding to the k^{th} sample of the DFT, $X[k]$, is given by $\omega = \frac{2\pi}{N}k$, where N is the dimension of the DFT. In our case, $N = 20000$ (2 seconds sampled at 10KHz), confirmed by displaying
```
>> length(xfreq)
```
Thus, the frequency corresponding to $k = 2000$ is $\omega = \frac{2\pi}{20000}2000 = 0.2\pi$.
Through sampling, the continuous-time angular frequency Ω is mapped to the discrete-time frequency $\omega = \Omega T_s$, where T_s is the sampling period. In our case, $\omega = 0.2\pi$ (the peak in $K = 2000$) corresponds to $\Omega = \frac{\omega}{T_s} = \omega F_s = 0.2\pi 10^4 = 2000\pi$, *i.e.*, to the frequency $f = \frac{\Omega}{2\pi} = 1000$Hz, as expected.
The peak at $k = 18000$ corresponds to $\omega = 1.8\pi$, which is the same as $\omega = -0.2\pi$, thus corresponds to the frequency that is symmetric to $\omega = 0.2\pi$, as always required when dealing with real signals as sums of complex exponentials.
We played the sound x and heard a "pure" sound, lasting 2 seconds:
```
>> soundsc(x,fs)
```

2. Signal $y[n]$:
```
>> y=atan((2*x+1)/4);
```
$Y[k]$, the DFT of $y[n]$:
```
>> yfreq=fft(y);
```
To interpret, we visualized the magnitude again:
```
>> plot(0:length(yfreq)-1,abs(yfreq))
```
We observed that $|Y[k]|$ has peaks for k multiple of 2000. This is an evidence that $y[n]$ is periodic, with fundamental frequency $\omega = 0.2\pi$ (as seen above, this is the frequency that corresponds to $k = 2000$). In fact, this was expected, since $y[n]$ was obtained through a memoryless transformation of $x[n]$, which is itself periodic, with fundamental frequency $\omega = 0.2\pi$.

3. $\omega(0) = \frac{2\pi}{5} \Leftrightarrow \frac{1}{\frac{1}{\omega_0}+k0} = \frac{2\pi}{5} \Leftrightarrow \omega_0 = \frac{2\pi}{5}\,, \quad \omega(10^4-1) = \frac{2\pi}{20} \Leftrightarrow \frac{1}{\frac{5}{2\pi}+k(10^4-1)} = \frac{2\pi}{20} \Leftrightarrow k = \frac{15}{2\pi(10^4-1)}$
Chirp signal:
```
>> w0=2*pi/5;
>> k=15/(2*pi*(1e4-1));
>> n=0:1e4-1;
>> x=cos(log(1+k*w0*n)/k);
```
Visualization of its spectrum (magnitude of the DFT):
```
>> xfreq=fft(x);
>> plot(0:length(xfreq)-1,abs(xfreq))
```
We observed that the energy of the signal is concentrated in the frequency region that corresponds roughly to the interval $500 < k < 2000$. Proceeding as in item **1**, we get that these values map to the frequencies $\frac{2\pi}{10^4}500 < \omega < \frac{2\pi}{10^4}2000 \Leftrightarrow \frac{2\pi}{20} < \omega < \frac{2\pi}{5}$. As expected, this agrees with the values chosen for the instantaneous frequency of the chirp signal, confirming that the DFT captures the frequency content of the signal. However, the DFT does not capture the time-evolution of the frequency content of the signal, *i.e.*, from the plot we only conclude that the entire signal segment has energy within the frequencies above but not how its frequency content changed across time.

4. Spectrogram of the chirp:
```
>> spectrogram(x,512,128)
```
We observed that the spectrogram captures the time-evolution of the frequency content of the signal. In

fact, we are now able to see that the signal started by having its energy around $\omega = 0.4\pi = \frac{2\pi}{5}$ and ended with energy concentrated at $\omega = 0.1\pi = \frac{2\pi}{20}$, as expected.

5.

```
>> specgramdemo
```

In fact, we observed the “layered” representation in two time segments. This means that, in those time segments, the spectrum of the signal has peaks at frequencies that are multiple of a given value. Similarly to what was pointed out in item **2.** above, this is an indication that the speech signal is approximately periodic (with fundamental frequency equal to that value) in those time-segments.