INSTITUTO SUPERIOR TÉCNICO

DSP 10/11 — Digital Signal Processing, 1st Test, April 8th, 2011

Show all your work on the exam pages and make sure you justify all your answers (results that are not explained or justified may count less, even if they are correct). Duration: 2 hours. Good luck!

1. Consider the causal LTI system for which the input $x[n]$ and the output $y[n]$ satisfy the difference equation

$$y[n] + \frac{1}{2}y[n-1] = 15x[n]\,.$$

a) Find its frequency response, $H(e^{j\omega})$.

b) Find its impulse response, $h[n]$.

c) Find the output $y[n]$ when the input is $x[n] = \cos\left(\frac{\pi}{2}n\right)$.

2. The system function of an LTI system is

$$H(z) = \frac{5}{1-2z^{-1}}\,, \qquad |z| > 2\,.$$

a) Is the system causal?

b) Is the system stable?

c) Find the output $y[n]$ when the input is $x[n] = u[n]$.

3. Consider three finite-length signals, $x_1[n]$, $x_2[n]$, and $x_3[n]$, that are zero outside the range $0 \le n \le 9$. We know the values of $x_1[n]$, the Fourier Transform (FT) of $x_2[n]$, and samples of FT of $x_3[n]$:

$$x_1[n] = \cos\left(\frac{3\pi}{5}n\right)\,, \qquad X_2\left(e^{j\omega}\right) = \frac{1-2^{10}e^{-j10\omega}}{1-2e^{-j\omega}}\,, \qquad X_3\left(e^{j\frac{2\pi}{5}k}\right) = 1\,,\ 0 \le k \le 4\,.$$

a) Compute and sketch $X_1[k]$, the 10-point DFT of $x_1[n]$.

b) Find $X_2[k]$, the 10-point DFT of $x_2[n]$.

c) Does the partial knowledge of $X_3\left(e^{j\omega}\right)$ uniquely determine $x_3[n]$? If it does, find $x_3[n]$; if not, illustrate with (at least two) distinct solutions for $x_3[n]$.

4. With the goal of analyzing a bandlimited continuous-time signal $x_c(t)$, whose highest frequency is 4KHz, we sample it, using the sampling frequency of 10KHz, obtaining $x[n]$. Then, we compute the localized FT of $x[n]$, *i.e.*, its spectrogram, as given by

$$X[n,k] = \sum_{m=0}^{49} x[n+m]e^{-j\frac{2\pi}{500}km}\,, \qquad 0 \le k \le 499\,.$$

a) To what continuous-time instant t does the index $n = 30000$ in $X[n,k]$ correspond?

b) To what frequency f (in Hz, *i.e.*, in continuous-time) does the index $k = 50$ in $X[n,k]$ correspond?

c) Is it possible, with this process, to identify the two components of the signal

$$x_c(t) = \cos\left(2\pi 1000t\right) + \cos\left(2\pi 1040t\right)?$$