The goal of this problem is to help you familiarizing with featureless motion estimation techniques, by using a simple, concrete, example.

We consider the (one-dimensional continuous) "images" $\mathbf{I}_1(x)$ and $\mathbf{I}_2(x)$, given by

$$\mathbf{I}_1(x) = \begin{cases} 1+x & \text{if } -1 \leq x \leq 0 \\ 1-x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad \mathbf{I}_2(x) = \begin{cases} 0.5+x & \text{if } -0.5 \leq x \leq 0.5 \\ 1.5-x & \text{if } 0.5 \leq x \leq 1.5 \\ 0 & \text{otherwise} \end{cases}.$$

As you can see, $\mathbf{I}_2$ is a translated version of $\mathbf{I}_1$. Our goal is to estimate the translation from the observations $\mathbf{I}_1(x)$ and $\mathbf{I}_2(x)$, $-\infty \leq x \leq +\infty$.

We thus consider the translational motion model $\mathbf{I}_2(x) = \mathbf{I}_1(x-p)$ and want to estimate p by using the iterative featureless motion estimation technique studied in [A].

1 Consider the initial estimate $\hat{p}_0 = 0$. Compute the result of the first iteration $\hat{p}_1$.

2 Discuss the convergence of $\hat{p}_n$ (either finding a general expression for $\hat{p}_n, n \geq 1$, or just providing insight, graphically, etc, over why $\lim_{n\to\infty} \hat{p}_n = 0.5$).

[A] "Multiresolution image alignment – lecture notes", P. Aguiar, IST, 2008.