(b) The inverse Haar transform is $\mathbf{F} = \mathbf{H}^T\mathbf{T}\mathbf{H}$, where $\mathbf{T}$ is the Haar transform of $\mathbf{F}$ and $\mathbf{H}^T$ is the matrix inverse of $\mathbf{H}$. Show that $\mathbf{H}_2^{-1} = \mathbf{H}_2^T$ and use it to compute the inverse Haar transform of the result in (a).

7.10 Compute the expansion coefficients of 2-tuple $[3, 2]^T$ for the following bases and write the corresponding expansions:

★**(a)** Basis $\varphi_0 = \left[1/\sqrt{2}, 1/\sqrt{2},\right]^T$ and $\varphi_1 = \left[1/\sqrt{2}, -1/\sqrt{2},\right]^T$ on $\mathbf{R}^2$, the set of real 2-tuples.

(b) Basis $\varphi_0 = [1, 0]^T$ and $\varphi_1 = [1, 1]^T$, and its dual, $\widetilde{\varphi}_0 = [1, -1]^T$ and $\widetilde{\varphi}_1 = [0, 1]^T$, on $\mathbf{R}^2$.

(c) Basis $\varphi_0 = [1, 0]^T$, $\varphi_1 = \left[-1/2, \sqrt{3}/2\right]^T$, and $\varphi_2 = \left[-1/2, -\sqrt{3}/2\right]^T$, and their duals, $\widetilde{\varphi}_i = 2\varphi_i/3$ for $i = \{0, 1, 2,\}$, on $\mathbf{R}^2$.

(*Hint*: Vector inner products must be used in place of the integral inner products of Section 7.2.1.)

7.11 Show that scaling function

$$\varphi(x) = \begin{cases} 1 & 0.25 \le x < 0.75 \\ 0 & \text{elsewhere} \end{cases}$$

does not satisfy the second requirement of a multiresolution analysis.

7.12 Write an expression for scaling space V_3 as a function of scaling function $\varphi(x)$. Use the Haar scaling function definition of Eq. (7.2-14) to draw the Haar V_3 scaling functions at translations $k = \{0, 1, 2\}$.

★**7.13** Draw wavelet $\psi_{3,3}(x)$ for the Haar wavelet function. Write an expression for $\psi_{3,3}(x)$ in terms of Haar scaling functions.

7.14 Suppose function $f(x)$ is a member of Haar scaling space V_3—that is, $f(x) \in V_3$. Use Eq. (7.2-22) to express V_3 as a function of scaling space V_0 and any required wavelet spaces. If $f(x)$ is 0 outside the interval $[0, 1)$, sketch the scaling and wavelet functions required for a linear expansion of $f(x)$ based on your expression.

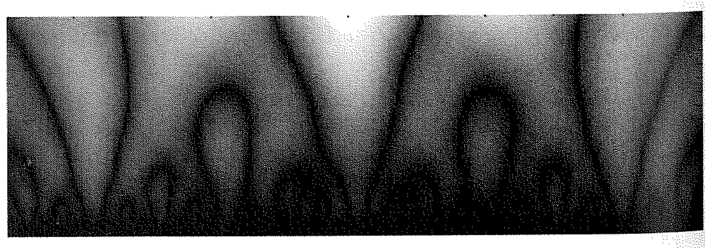
7.15 Compute the first four terms of the wavelet series expansion of the function used in Example 7.7 with starting scale $j_0 = 1$. Write the resulting expansion in terms of the scaling and wavelet functions involved. How does your result compare to the example, where the starting scale was $j_0 = 0$?

7.16 The DWT in Eqs. (7.3-5) and (7.3-6) is a function of starting scale j_0.

(a) Recompute the one-dimensional DWT of function $f(n) = \{1, 4, -3, 0\}$ for $0 \le n \le 3$ in Example 7.8 with $j_0 = 1$ (rather than 0).

(b) Use the result from (a) to compute $f(1)$ from the transform values.

★**7.17** What does the following continuous wavelet transform reveal about the one-dimensional function upon which it was based?



7.18 **(a)** The continuous wavelet transform of Problem 7.17 is computer generated. The function upon which it is based was first sampled at discrete intervals. What is continuous about the transform—or what distinguishes it from the discrete wavelet transform of the function?

★**(b)** Under what circumstances is the DWT a better choice than the CWT? Are there times when the CWT is better than the DWT?

★**7.19** Draw the FWT filter bank required to compute the transform in Problem 7.16. Label all inputs and outputs with the appropriate sequences.

7.20 The computational complexity of an M-point fast wavelet transform is $O(M)$. That is, the number of operations is proportional M. What determines the constant of proportionality?

7.21 ★**(a)** If the input to the three-scale FWT filter bank of Fig. 7.30(a) is the Haar scaling function $\varphi(n) = 1$ for $n = 0, 1, \ldots, 7$ and 0 elsewhere, what is the resulting transform with respect to Haar wavelets?

(b) What is the transform if the input is the corresponding Haar wavelet function $\psi(n) = \{1, 1, 1, 1, -1, -1, -1, -1\}$ for $n = 0, 1, \ldots, 7$?

(c) What input sequence produces transform $\{0, 0, 0, 0, 0, 0, B, 0\}$ with nonzero coefficient $W_\psi(2, 2) = B$?

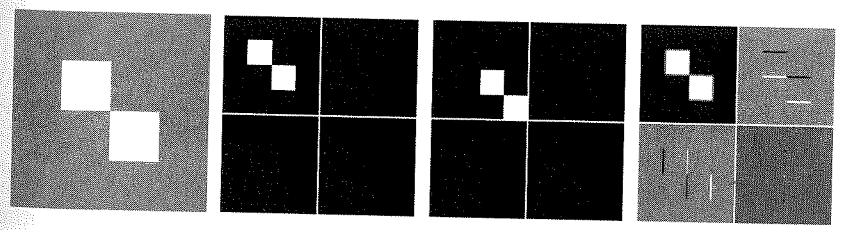
★**7.22** The two-dimensional fast wavelet transform is similar to the pyramidal coding scheme of Section 7.2.1. How are they similar? Given the three-scale wavelet transform in Fig. 7.10(a), how would you construct the corresponding approximation pyramid? How many levels would it have?

7.23 Compute the two-dimensional wavelet transform with respect to Haar wavelets of the 2×2 image in Problem 7.9. Draw the required filter bank and label all inputs and outputs with the proper arrays.

★**7.24** In the Fourier domain

$$f(x - x_0, y - y_0) \Leftrightarrow F(\mu, v)e^{-2\pi(\mu x_0/M + v y_0/N)}$$

and translation does not affect the display of $|F(\mu, v)|$. Using the following sequence of images, explain the translation property of wavelet transforms. The leftmost image contains two 32×32 white squares centered on a 128×128 gray background. The second image (from the left) is its single-scale wavelet transform with respect to Haar wavelets. The third is the wavelet transform of the original image after shifting it 32 pixels to the right and downward, and the final (rightmost) image is the wavelet transform of the original image after it has been shifted one pixel to the right and downward.



7.25 The following table shows the Haar wavelet and scaling functions for a four-scale fast wavelet transform. Sketch the additional basis functions needed for a full three-scale packet decomposition. Give the mathematical expression or expressions for determining them. Then order the basis functions according to frequency content and explain the results.