5.21   A certain X-ray imaging geometry produces a blurring degradation that can be modeled as the convolution of the sensed image with the spatial, circularly symmetric function

$$h(x, y) = \frac{x^2 + y^2 - 2\sigma^2}{\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Assuming continuous variables, show that the degradation in the frequency domain is given by the expression

$$H(u, v) = -8\pi^2\sigma^2(u^2 + v^2)e^{-2\pi^2\sigma^2(u^2+v^2)}$$

(*Hint:* Refer to Section 4.9.4, entry 13 in Table 4.3, and Problem 4.26.)

★ **5.22**   Using the transfer function in Problem 5.21, give the expression for a Wiener filter, assuming that the ratio of power spectra of the noise and undegraded signal is a constant.

5.23   Using the transfer function in Problem 5.21, give the resulting expression for the constrained least squares filter.

5.24   Assume that the model in Fig. 5.1 is linear and position invariant and that the noise and image are uncorrelated. Show that the power spectrum of the output is

$$|G(u, v)|^2 = |H(u, v)|^2|F(u, v)|^2 + |N(u, v)|^2$$

Refer to Eqs. (5.5-17) and (4.6-18).

5.25   Cannon [1974] suggested a restoration filter $R(u, v)$ satisfying the condition

$$|\hat{F}(u, v)|^2 = |R(u, v)|^2|G(u, v)|^2$$

and based on the premise of forcing the power spectrum of the restored image, $|\hat{F}(u, v)|^2$, to equal the power spectrum of the original image, $|F(u, v)|^2$. Assume that the image and noise are uncorrelated.

★ **(a)** Find $R(u, v)$ in terms of $|F(u, v)|^2$, $|H(u, v)|^2$, and $|N(u, v)|^2$. [*Hint:* Refer to Fig. 5.1, Eq. (5.5-17), and Problem 5.24.]

(b) Use your result in (a) to state a result in the form of Eq. (5.8-2).

5.26   An astronomer working with a large-scale telescope observes that her images are a little blurry. The manufacturer tells the astronomer that the unit is operating within specifications. The telescope lenses focus images onto a high-resolution, CCD imaging array, and the images are then converted by the telescope electronics into digital images. Trying to improve the situation by conducting controlled lab experiments with the lenses and imaging sensors is not possible due to the size and weight of the telescope components. The astronomer, having heard about your success as an image processing expert, calls you to help her formulate a digital image processing solution for sharpening the images a little more. How would you go about solving this problem, given that the only images you can obtain are images of stellar bodies?

★ **5.27**   A professor of archeology doing research on currency exchange practices during the Roman Empire recently became aware that four Roman coins crucial to his research are listed in the holdings of the British Museum in London. Unfortunately, he was told after arriving there that the coins recently had been stolen. Further research on his part revealed that the museum keeps photographs of

every item for which it is responsible. Unfortunately, the photos of the coins in question are blurred to the point where the date and other small markings are not readable. The cause of the blurring was the camera being out of focus when the pictures were taken. As an image processing expert and friend of the professor, you are asked as a favor to determine whether computer processing can be utilized to restore the images to the point where the professor can read the markings. You are told that the original camera used to take the photos is still available, as are other representative coins of the same era. Propose a step-by-step solution to this problem.

5.28   Sketch the Radon transform of the following square images. Label quantitatively all the important features of your sketches. Figure (a) consists of one dot in the center, and (b) has two dots along the diagonal. Describe your solution to (c) by an intensity profile. Assume a parallel-beam geometry.

(a)   ★(b)   (c)
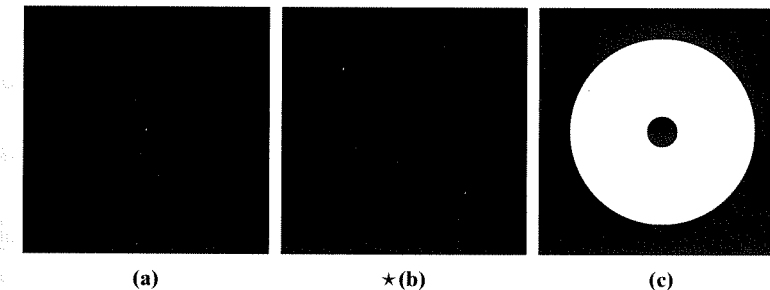
5.29   Show that the Radon transform [Eq. (5.11-3)] of the Gaussian shape $f(x, y) = A \exp(-x^2 - y^2)$ is $g(\rho, \theta) = A\sqrt{\pi} \exp(-\rho^2)$. (*Hint:* Refer to Example 5.17, where we used symmetry to simplify integration.)

5.30 ★ **(a)** Show that the Radon transform [Eq. (5.11-3)] of the unit impulse $\delta(x, y)$ is a straight vertical line in the $\rho\theta$-plane passing through the origin.

(b) Show that the radon transform of the impulse $\delta(x - x_0, y - y_0)$ is a sinusoidal curve in the $\rho\theta$-plane.

5.31   Prove the validity of the following properties of the Radon transform [Eq. (5.11-3)]:

★ **(a)** Linearity: The Radon transform is a linear operator. (See Section 2.6.2 regarding the definition of linear operators.)

(b) Translation property: The radon transform of $f(x - x_0, y - y_0)$ is $g(\rho - x_0 \cos_\theta - y_0 \sin_\theta, \theta)$.

★ **(c)** Convolution property: Show that the Radon transform of the convolution of two functions is equal to the convolution of the Radon transforms of the two functions.

5.32   Provide the steps leading from Eq. (5.11-14) to (5.11-15). You will need to use the property $G(\omega, \theta + 180°) = G(-\omega, \theta)$.

★ **5.33**   Prove the validity of Eq. (5.11-25).

5.34   Prove the validity of Eq. (5.11-27).