

was split as two constants $1/\sqrt{MN}$ in front of both the forward and inverse transforms. How can you find where the term(s) is (are) included if this information is not available in the documentation?

- 4.16 ★(a) Prove the validity of the translation property in Eq. (4.6-3).
(b) Prove the validity of Eq. (4.6-4).

- 4.17 You can infer from Problem 4.3 that $1 \Leftrightarrow \delta(\mu, \nu)$ and $\delta(t, z) \Leftrightarrow 1$. Use the first of these properties and the translation property in Table 4.3 to show that the Fourier transform of the continuous function $f(t, z) = A \sin(2\pi\mu_0 t + 2\pi\nu_0 z)$ is

$$F(\mu, \nu) = \frac{j}{2} [\delta(\mu + \mu_0, \nu + \nu_0) - \delta(\mu - \mu_0, \nu - \nu_0)]$$

- 4.18 Show that the DFT of the discrete function $f(x, y) = 1$ is

$$\mathfrak{F}\{1\} = \delta(u, v) = \begin{cases} 1 & \text{if } u = v = 0 \\ 0 & \text{otherwise} \end{cases}$$

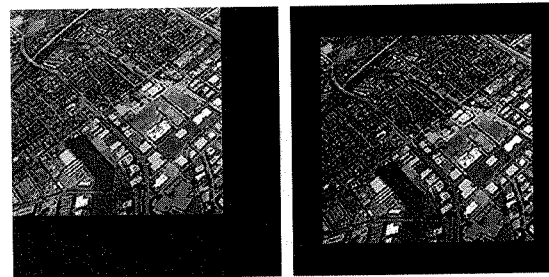
- 4.19 Show that the DFT of the discrete function $f(x, y) = \sin(2\pi u_0 x + 2\pi v_0 y)$ is

$$F(u, v) = \frac{j}{2} [\delta(u + Mu_0, v + Nv_0) - \delta(u - Mu_0, v - Nv_0)]$$

- 4.20 The following problems are related to the properties in Table 4.1.

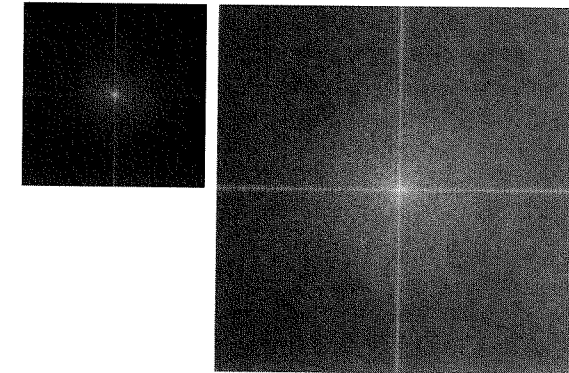
- ★(a) Prove the validity of property 2.
- ★(b) Prove the validity of property 4.
- (c) Prove the validity of property 5.
- ★(d) Prove the validity of property 7.
- (e) Prove the validity of property 9.
- (f) Prove the validity of property 10.
- ★(g) Prove the validity of property 11.
- (h) Prove the validity of property 12.
- (i) Prove the validity of property 13.

- ★4.21 The need for image padding when filtering in the frequency domain was discussed in Section 4.6.6. We showed in that section that images needed to be padded by appending zeros to the ends of rows and columns in the image (see the following image on the left). Do you think it would make a difference if we



centered the image and surrounded it by a border of zeros instead (see image on the right), but without changing the total number of zeros used? Explain.

- ★4.22 The two Fourier spectra shown are of the same image. The spectrum on the left corresponds to the original image, and the spectrum on the right was obtained after the image was padded with zeros. Explain the significant increase in signal strength along the vertical and horizontal axes of the spectrum shown on the right.



- 4.23 You know from Table 4.2 that the dc term, $F(0, 0)$, of a DFT is proportional to the average value of its corresponding spatial image. Assume that the image is of size $M \times N$. Suppose that you pad the image with zeros to size $P \times Q$, where P and Q are given in Eqs. (4.6-31) and (4.6-32). Let $F_p(0, 0)$ denote the dc term of the DFT of the padded function.

- ★(a) What is the ratio of the average values of the original and padded images?
- (b) Is $F_p(0, 0) = F(0, 0)$? Support your answer mathematically.

- 4.24 Prove the periodicity properties (entry 8) in Table 4.2.

- 4.25 The following problems are related to the entries in Table 4.3.

- ★(a) Prove the validity of the discrete convolution theorem (entry 6) for the 1-D case.

- (b) Repeat (a) for 2-D.

- ★(c) Prove the validity of entry 7.

- ★(d) Prove the validity of entry 12.

(Note: Problems 4.18, 4.19, and 4.31 are related to Table 4.3 also.)

- 4.26 (a) Show that the Laplacian of a continuous function $f(t, z)$ of continuous variables t and z satisfies the following Fourier transform pair [see Eq. (3.6-3) for a definition of the Laplacian]:

$$\nabla^2 f(t, z) \Leftrightarrow -4\pi^2(\mu^2 + \nu^2)F(\mu, \nu)$$

[Hint: Study entry 12 in Table 4.3 and see Problem 4.25(d).]

- ★(b) The preceding closed form expression is valid only for continuous variables. However, it can be the basis for implementing the Laplacian in the discrete frequency domain using the $M \times N$ filter

$$H(u, v) = -4\pi^2(u^2 + v^2)$$