★3.19   **(a)** Develop a procedure for computing the median of an $n \times n$ neighborhood.

   (b) Propose a technique for updating the median as the center of the neighborhood is moved from pixel to pixel.

3.20   **(a)** In a character recognition application, text pages are reduced to binary form using a thresholding transformation function of the form shown in Fig. 3.2(b). This is followed by a procedure that thins the characters until they become strings of binary 1s on a background of 0s. Due to noise, the binarization and thinning processes result in broken strings of characters with gaps ranging from 1 to 3 pixels. One way to "repair" the gaps is to run an averaging mask over the binary image to blur it, and thus create bridges of nonzero pixels between gaps. Give the (odd) size of the smallest averaging mask capable of performing this task.

   (b) After bridging the gaps, it is desired to threshold the image in order to convert it back to binary form. For your answer in (a), what is the minimum value of the threshold required to accomplish this, without causing the segments to break up again?

★3.21   The three images shown were blurred using square averaging masks of sizes $n = 23, 25$, and 45, respectively. The vertical bars on the left lower part of (a) and (c) are blurred, but a clear separation exists between them. However, the bars have merged in image (b), in spite of the fact that the mask that produced this image is significantly smaller than the mask that produced image (c). Explain the reason for this.

(a)   (b)   (c)
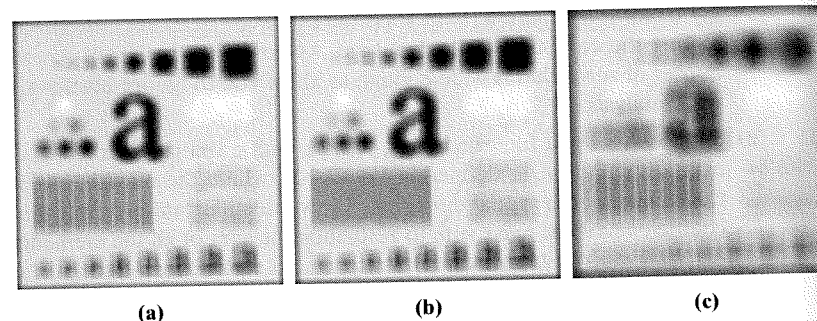
3.22   Consider an application such as the one shown in Fig. 3.34, in which it is desired to eliminate objects smaller than those enclosed by a square of size $q \times q$ pixels. Suppose that we want to reduce the average intensity of those objects to one-tenth of their original average value. In this way, those objects will be closer to the intensity of the background and they can then be eliminated by thresholding. Give the (odd) size of the smallest averaging mask that will accomplish the desired reduction in average intensity in only one pass of the mask over the image.

3.23   In a given application an averaging mask is applied to input images to reduce noise, and then a Laplacian mask is applied to enhance small details. Would the result be the same if the order of these operations were reversed?

★3.24   Show that the Laplacian defined in Eq. (3.6-3) is isotropic (invariant to rotation). You will need the following equations relating coordinates for axis rotation by an angle θ:

$$x = x' \cos \theta - y' \sin \theta$$
$$y = x' \sin \theta + y' \cos \theta$$

where (x, y) are the unrotated and (x', y') are the rotated coordinates.

★3.25   You saw in Fig. 3.38 that the Laplacian with a -8 in the center yields sharper results than the one with a -4 in the center. Explain the reason in detail.

3.26   With reference to Problem 3.25,

   (a) Would using a larger "Laplacian-like" mask, say, of size 5×5 with a -24 in the center, yield an even sharper result? Explain in detail.

   (b) How does this type of filtering behave as a function of mask size?

3.27   Give a 3×3 mask for performing unsharp masking in a single pass through an image. Assume that the average image is obtained using the filter in Fig. 3.32(a).

★3.28   Show that subtracting the Laplacian from an image is proportional to unsharp masking. Use the definition for the Laplacian given in Eq. (3.6-6).

3.29   **(a)** Show that the magnitude of the gradient given in Eq. (3.6-11) is an isotropic operation. (See Problem 3.24.)

   (b) Show that the isotropic property is lost in general if the gradient is computed using Eq. (3.6-12).

3.30   A CCD TV camera is used to perform a long-term study by observing the same area 24 hours a day, for 30 days. Digital images are captured and transmitted to a central location every 5 minutes. The illumination of the scene changes from natural daylight to artificial lighting. At no time is the scene without illumination, so it is always possible to obtain an image. Because the range of illumination is such that it is always in the linear operating range of the camera, it is decided not to employ any compensating mechanisms on the camera itself. Rather, it is decided to use image processing techniques to post-process, and thus normalize, the images to the equivalent of constant illumination. Propose a method to do this. You are at liberty to use any method you wish, but state clearly all the assumptions you made in arriving at your design.

3.31   Show that the crossover point in Fig. 3.46(d) is given by $b = (a + c)/2$.

3.32   Use the fuzzy set definitions in Section 3.8.2 and the basic membership functions in Fig. 3.46 to form the membership functions shown below.

★ (a)   (b)   (c)