3.7 Suppose that a digital image is subjected to histogram equalization. Show that a second pass of histogram equalization (on the histogram-equalized image) will produce exactly the same result as the first pass.

3.8 In some applications it is useful to model the histogram of input images as Gaussian probability density functions of the form

$$p_r(r) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(r-m)^2}{2\sigma^2}}$$

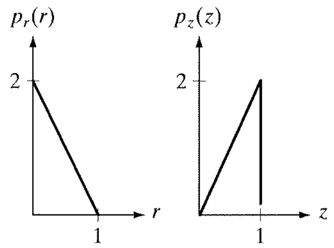
where m and σ are the mean and standard deviation of the Gaussian PDF. The approach is to let m and σ be measures of average intensity and contrast of a given image. What is the transformation function you would use for histogram equalization?

★3.9 Assuming continuous values, show by example that it is possible to have a case in which the transformation function given in Eq. (3.3-4) satisfies conditions (a) and (b) in Section 3.3.1, but its inverse may fail condition (a′).

3.10 **(a)** Show that the discrete transformation function given in Eq. (3.3-8) for histogram equalization satisfies conditions (a) and (b) in Section 3.3.1.

★(b) Show that the inverse discrete transformation in Eq. (3.3-9) satisfies conditions (a′) and (b) in Section 3.3.1 only if none of the intensity levels $r_k, k = 0, 1, \ldots, L - 1$, are missing.

3.11 An image with intensities in the range [0, 1] has the PDF $p_r(r)$ shown in the following diagram. It is desired to transform the intensity levels of this image so that they will have the specified $p_z(z)$ shown. Assume continuous quantities and find the transformation (in terms of r and z) that will accomplish this.



★3.12 Propose a method for updating the local histogram for use in the local enhancement technique discussed in Section 3.3.3.

3.13 Two images, $f(x, y)$ and $g(x, y)$, have histograms h_f and h_g. Give the conditions under which you can determine the histograms of

★(a) $f(x, y) + g(x, y)$

(b) $f(x, y) - g(x, y)$

(c) $f(x, y) \times g(x, y)$

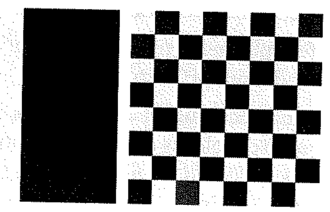
(d) $f(x, y) \div g(x, y)$

in terms of h_f and h_g. Explain how to obtain the histogram in each case.

3.14 The images shown on the next page are quite different, but their histograms are the same. Suppose that each image is blurred with a 3×3 averaging mask.

(a) Would the histograms of the blurred images still be equal? Explain.

(b) If your answer is no, sketch the two histograms.



3.15 The implementation of linear spatial filters requires moving the center of a mask throughout an image and, at each location, computing the sum of products of the mask coefficients with the corresponding pixels at that location (see Section 3.4). A lowpass filter can be implemented by setting all coefficients to 1, allowing use of a so-called *box-filter* or *moving-average* algorithm, which consists of updating only the part of the computation that changes from one location to the next.

★(a) Formulate such an algorithm for an $n \times n$ filter, showing the nature of the computations involved and the scanning sequence used for moving the mask around the image.

(b) The ratio of the number of computations performed by a brute-force implementation to the number of computations performed by the box-filter algorithm is called the *computational advantage*. Obtain the computational advantage in this case and plot it as a function of n for $n > 1$. The $1/n^2$ scaling factor is common to both approaches, so you need not consider it in obtaining the computational advantage. Assume that the image has an outer border of zeros that is wide enough to allow you to ignore border effects in your analysis.

3.16 ★(a) Suppose that you filter an image, $f(x, y)$, with a spatial filter mask, $w(x, y)$, using convolution, as defined in Eq. (3.4-2), where the mask is smaller than the image in both spatial directions. Show the important property that, if the coefficients of the mask sum to zero, then the sum of all the elements in the resulting convolution array (filtered image) will be zero also (you may ignore computational inaccuracies). Also, you may assume that the border of the image has been padded with the appropriate number of zeros.

(b) Would the result to (a) be the same if the filtering is implemented using correlation, as defined in Eq. (3.4-1)?

3.17 Discuss the limiting effect of repeatedly applying a 3×3 lowpass spatial filter to a digital image. You may ignore border effects.

3.18 ★(a) It was stated in Section 3.5.2 that isolated clusters of dark or light (with respect to the background) pixels whose area is less than one-half the area of a median filter are eliminated (forced to the median value of the neighbors) by the filter. Assume a filter of size $n \times n$, with n odd, and explain why this is so.

(b) Consider an image having various sets of pixel clusters. Assume that all points in a cluster are lighter or darker than the background (but not both simultaneously in the same cluster), and that the area of each cluster is less than or equal to $n^2/2$. In terms of n, under what condition would one or more of these clusters cease to be isolated in the sense described in part (a)?