11.8 Draw the medial axis of

★(a) A circle
★(b) A square
(c) A rectangle
(d) An equilateral triangle

11.9 For each of the figures shown,

★(a) Discuss the action taken at point p by Step 1 of the skeletonizing algorithm presented in Section 11.1.7.

(b) Repeat for Step 2 of the algorithm. Assume that $p = 1$ in all cases.

1	1	0
1	p	0
1	1	0

0	0	0
1	p	0
0	0	0

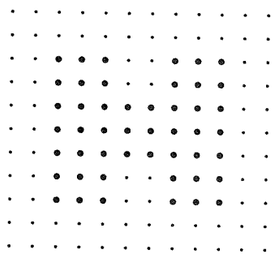
0	1	0
1	p	1
0	1	0

1	1	0
0	p	1
0	0	0

11.10 With reference to the skeletonizing algorithm in Section 11.1.7, what would the figure shown look like after

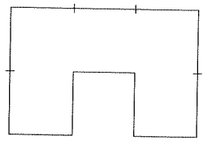
★(a) One pass of Step 1 of the algorithm?

(b) One pass of Step 2 (on the result of Step 1, not the original image)?



11.11 **★(a)** What is the order of the shape number for the figure shown?

(b) Obtain the shape number.



11.12 The procedure discussed in Section 11.2.3 for using Fourier descriptors consists of expressing the coordinates of a contour as complex numbers, taking the DFT of these numbers, and keeping only a few components of the DFT as descriptors of the boundary shape. The inverse DFT is then an approximation to the original contour. What class of contour shapes would have a DFT consisting of real numbers and how would the axis system in Fig. 11.19 have to be set up to obtain these real numbers?

11.13 Show that if you use only two Fourier descriptors ($u = 0$ and $u = 1$) to reconstruct a boundary with Eq. (11.2-5), the result will always be a circle. (*Hint:* Use the parametric representation of a circle in the complex plane and express the equation of a circle in polar coordinates.)

★11.14 Give the smallest number of statistical moment descriptors needed to differentiate between the signatures of the figures shown in Fig. 11.10.

11.15 Give two boundary shapes that have the same mean and third statistical moment descriptors, but different second moments.

★11.16 Propose a set of descriptors capable of differentiating between the shapes of the characters 0, 1, 8, 9, and X. (*Hint:* Use topological descriptors in conjunction with the convex hull.)

11.17 Consider a binary image of size 200 × 200 pixels, with a vertical black band extending from columns 1 to 99 and a vertical white band extending from columns 100 to 200.

(a) Obtain the co-occurrence matrix of this image using the position operator "one pixel to the right."

★(b) Normalize this matrix so that its elements become probability estimates, as explained in Section 11.3.1.

(c) Use your matrix from (b) to compute the six descriptors in Table 11.3.

11.18 Consider a checkerboard image composed of alternating black and white squares, each of size $m \times m$. Give a position operator that would yield a diagonal co-occurrence matrix.

11.19 Obtain the gray-level co-occurrence matrix of a 5 × 5 image composed of a checkerboard of alternating 1s and 0s if

★(a) the position operator Q is defined as "one pixel to the right," and

(b) the position operator Q is defined as "two pixels to the right."

Assume that the top left pixel has value 0.

11.20 Prove the validity of Eqs. (11.4-7), (11.4-8), and (11.4-9).

★11.21 It was mentioned in Example 11.13 that a credible job could be done of reconstructing approximations to the six original images by using only the two principal-component images associated with the largest eigenvalues. What would be the mean square error incurred in doing so? Express your answer as a percentage of the maximum possible error.

11.22 For a set of images of size 64 × 64, assume that the covariance matrix given in Eq. (11.4-9) turns out to be the identity matrix. What would be the mean square error between the original images and images reconstructed using Eq. (11.4-11) with only half of the original eigenvectors?

★11.23 Under what conditions would you expect the major axes of a boundary, defined in Section 11.2.1, to be equal to the eigen axes of that boundary?

11.24 Give a spatial relationship and corresponding tree representation for a checkerboard pattern of black and white squares. Assume that the top left element is black and that the root of the tree corresponds to that element. Your tree can have no more than two branches emanating from each node.

★11.25 You are contracted to design an image processing system for detecting imperfections on the inside of certain solid plastic wafers. The wafers are examined using an X-ray imaging system, which yields 8-bit images of size 512 × 512. In the absence of imperfections, the images appear "bland," having a mean intensity of 100 and variance of 400. The imperfections appear as bloblike regions in which about 70% of the pixels have excursions in intensity of 50 intensity levels or less about a