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IMAGE MOTION ESTIMATION – CONVERGENCE AND ERROR ANALYSIS

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ABSTRACT

The paper computes the reliability of estimates of image motion parameters. The use of such measures of reliability to weight motion estimates improves significantly the performance of motion analysis tasks such as the recovery of 3D structure [1, 2]. The paper relates both the estimation error variance and the stability of the estimation algorithm with the spatial gradient of the image brightness pattern. We illustrate the predictions of our expressions with several image brightness patterns.

1. INTRODUCTION

The 2D motion of the image brightness pattern across an image sequence contains significant information about the 3D motion of the camera, the 3D structure of the scene, and the presence of independently moving objects. As a consequence, *motion analysis* finds a large number of applications in areas that include digital video, robotics, and virtual reality. Estimating the 2D image motion is a crucial step in any motion analysis task.

This paper deals with the estimation of the 2D motion of the brightness pattern in the image plane. This problem has been widely addressed in the recent past. The original contribution of this paper is the development of reliability measures to the estimates of the parameters describing the image motion. We relate these measures to the spatial gradient of the brightness pattern. The availability of these reliability measures increases the performance and decreases the computational burden in motion analysis tasks. The gain in computational simplicity comes from discarding those regions whose brightness pattern leads to very inaccurate 2D motion estimates. The increase in performance of the motion analysis task is due to weighting differently the several 2D displacements, according to their different reliability. For example, it has been shown that weighting more the trajectories corresponding to "sharp" features than the trajectories corresponding to features with smooth textures leads to better 3D reconstructions [1, 2]. Reference [3] is one of the few papers dealing with motion estimation accuracy. The goal of these authors was the development of feature selection criteria; they don't quantify the estimation error, as we do in this paper.

A common technique for image motion estimation is to parameterize the 2D motion and to minimize the brightness square difference by using a Gauss-Newton iterative algorithm. In section 2, we discuss the behavior of this motion estimation algorithm. In section 3, we study the estimation error. We show that the estimator is unbiased and we develop the expression for the error covariance matrix. Section 4 particularizes the study of sections 2 and 3 to the translational motion model. Section 5 concludes the paper.

2. MOTION ESTIMATION

Consider the pair of images $\{\mathbf{I}_1, \mathbf{I}_2\}$. Our goal is the estimation of the motion of the brightness pattern between images $\mathbf{I}_1$ and $\mathbf{I}_2$ in a given region $\mathcal{R}$ of the image plane. We parameterize the 2D motion in terms of the vector $\mathbf{p}$. The estimate $\widehat{\mathbf{p}}$ of $\mathbf{p}$ is

$$\widehat{\mathbf{p}} = \arg\min_{\mathbf{p}} \iint_{\mathcal{R}} e^2(\mathbf{p}; x, y)\, dx\, dy, \qquad \text{where} \tag{1}$$

$$e(\mathbf{p}; x, y) = \mathbf{I}_1(x, y) - \mathbf{I}_2\Big(x + d_x(\mathbf{p}; x, y), y + d_y(\mathbf{p}; x, y)\Big).$$

The displacement of the pixel (x, y) between images $\mathbf{I}_1$ and $\mathbf{I}_2$ is denoted by $\mathbf{d}(\mathbf{p}; x, y) = [d_x(\mathbf{p}; x, y), d_y(\mathbf{p}; x, y)]^T$.

The minimization in expression (1) is commonly accomplished by using a Gauss-Newton method – the estimate $\widehat{\mathbf{p}}$ is computed by refining a previous estimate $\mathbf{p}_0$, $\widehat{\mathbf{p}} = \mathbf{p}_0 + \widehat{\boldsymbol{\delta}_p}$. The iterative updating $\widehat{\boldsymbol{\delta}_p}$ is given by the solution of the following linear system, see [4],

$$\boldsymbol{\Gamma}_{\mathcal{R}}(\mathbf{p}_0)\, \widehat{\boldsymbol{\delta}_p} = \boldsymbol{\gamma}_{\mathcal{R}}(\mathbf{p}_0), \tag{3}$$

$$\boldsymbol{\Gamma}_{\mathcal{R}}(\mathbf{p}_0) = \iint_{\mathcal{R}} \nabla_{\mathbf{p}} \mathbf{d}^T(\mathbf{p}_0) \mathbf{i}_{xy} \mathbf{i}_{xy}^T \nabla_{\mathbf{p}} \mathbf{d}(\mathbf{p}_0)\, dx\, dy, \tag{4}$$

$$\boldsymbol{\gamma}_{\mathcal{R}}(\mathbf{p}_0) = -\iint_{\mathcal{R}} i_t(\mathbf{p}_0) \nabla_{\mathbf{p}} \mathbf{d}^T(\mathbf{p}_0) \mathbf{i}_{xy}\, dx\, dy, \tag{5}$$

where we omitted the dependence of the integrands on (x, y) for simplicity. The vector $\mathbf{i}_{xy}(x, y)$ is defined as $\mathbf{i}_{xy}(x, y) = [i_x(x, y), i_y(x, y)]^T$, where $i_x(x, y)$ and $i_y(x, y)$ are the spatial derivatives computed from the reference image $\mathbf{I}_1(x, y)$, $i_t(\mathbf{p}_0; x, y)$ is the temporal derivative, and the matrix $\nabla_{\mathbf{p}} \mathbf{d}^T(\mathbf{p}_0; x, y)$ is defined as $\nabla_{\mathbf{p}} \mathbf{d}^T(\mathbf{p}_0; x, y) = [\nabla_{\mathbf{p}} d_x(\mathbf{p}_0; x, y), \nabla_{\mathbf{p}} d_y(\mathbf{p}_0; x, y)]$.

In order to obtain a reliable convergence of the Gauss-Newton method, the equation system (3) must be well conditioned, i.e., the matrix $\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0)$, given by (4), must be well conditioned with respect to inversion. A widely used measure for the sensitivity of the solution of the linear system is the *condition number* of the square matrix involved. The relative error of the solution of a linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$ is approximated by the condition number $k(\mathbf{A})$ of the matrix $\mathbf{A}$ times the relative errors in $\mathbf{A}$ and $\mathbf{b}$. The condition number depends on the underlying norm used to measure the error. With the common choice of the matrix 2-norm, the condition number of a matrix is given by the quotient of the largest singular value by the smallest singular value. Since the matrix $\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0)$ is symmetric and semi-positive definite, their eigenvalues are positive real and coincide with the singular values. The sensitivity of the iterates of the motion estimation algorithm are then measured by

$$k(\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0)) = \frac{\lambda_1(\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0))}{\lambda_N(\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0))}, \tag{6}$$

where $\lambda_1(\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0))$ is the largest eigenvalue of $\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0)$ and $\lambda_N(\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0))$ is its smallest eigenvalue.

If the condition number $k(\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0))$ is large, the matrix $\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0)$ is said to be ill-conditioned. In this case, the Gauss-Newton iterates are very sensitive to the noise and the process can not be guaranteed to converge.

3. ESTIMATION ERROR

We study the statistics of the error in estimating the vector $\mathbf{p}$ of motion parameters. Our analysis is local, in the sense that we assume small deviations between the true value of the vector of motion parameters and its estimate. The statistics that we obtain are valid in practice as good approximations to the real statistics if the estimation problem is well conditioned, i.e., if the observations, regardless of the noise level, contain "enough information" to estimate the desired parameters (this imprecise definition can be made precise in terms of the usual signal to noise ratio parameter). This situation is the one in which we are interested in because in practical applications we only use the motion estimates when the corresponding estimation problem is well conditioned in the sense discussed in the previous section.

We denote the actual value of the vector of motion parameters by $\mathbf{p}_a$. The estimate $\widehat{\mathbf{p}}$ is written in terms of a small deviation relative to the actual $\mathbf{p}_a$. By proceeding in a similar way as done for the derivation of the estimation algorithm [4], we obtain,

$$\widehat{\mathbf{p}} = \mathbf{p}_a + \mathbf{\Gamma}_{\mathcal{R}}^{-1}(\mathbf{p}_a)\,\boldsymbol{\gamma}_{\mathcal{R}}(\mathbf{p}_a), \tag{7}$$

where, we recall, the matrix $\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_a)$ and the vector $\boldsymbol{\gamma}_{\mathcal{R}}(\mathbf{p}_a)$ are given by expressions (4) and (5) with $\mathbf{p}_a$ instead of $\mathbf{p}_0$. The random variable $\widehat{\mathbf{p}}$ in expression (7) is a non-linear function of the image derivatives $\{i_t(\mathbf{p}_a), \mathbf{i}_{xy} = [i_x, i_y]^T\}$.

The derivatives i_t and $\mathbf{i}_{xy}$ are random variables – they are noisy versions of the actual values of the scene brightness derivatives. The actual value of $i_t(\mathbf{p}_a)$ is $i_{ta}(\mathbf{p}_a) = 0$ because $\mathbf{p}_a$ is the actual value of the vector of motion parameters. Since the image noise is zero mean, the noise corrupting the derivatives i_t, i_x, and i_y is also zero mean. Furthermore, the noise corrupting the temporal derivative i_t is white because the image noise is white. We denote by σ_t^2 the variance of the noise corrupting i_t.

We find the expected value of the estimate $\widehat{\mathbf{p}}$ by computing the mean of expression (7) with respect to the noise of the image derivatives. For small deviations, the first-order approximation of $\mathrm{E}\{\widehat{\mathbf{p}}\}$ is given by the value of expression (7) evaluated at the mean values of the random variables $i_t(\mathbf{p}_a)$ and $\mathbf{i}_{xy}$. Since the mean of $i_t(\mathbf{p}_a)$ is zero, we get $\boldsymbol{\gamma}_{\mathcal{R}}(\mathbf{p}_a) = \mathbf{0}$ and the mean of the estimate $\widehat{\mathbf{p}}$ is

$$\mathrm{E}\left\{\widehat{\mathbf{p}}\right\} = \mathbf{p}_a + \mathrm{E}\left\{\boldsymbol{\delta}_p\right\} = \mathbf{p}_a. \tag{8}$$

Expression (8) states that, to first-order approximation, the estimate $\widehat{\mathbf{p}}$ is unbiased.

The covariance matrix of the estimating error is denoted by $\mathbf{\Sigma}_p$. The first-order approximation of $\mathbf{\Sigma}_p$ can be expressed in terms of the partial derivatives of the estimate $\widehat{\mathbf{p}}$ with respect to the random variables involved, i.e., with respect to i_t, i_x, and i_y, and to the variances of those random variables [4]. From expressions (7), (4), and (5), we compute the partial derivatives of $\widehat{\mathbf{p}}$ with respect to the random variables $i_t(x,y)$, $i_x(x,y)$, and $i_y(x,y)$. Using the fact that the noise corrupting i_t is white and noting that the derivatives $\frac{\partial \widehat{\mathbf{p}}}{\partial i_x(x,y)}$ and $\frac{\partial \widehat{\mathbf{p}}}{\partial i_y(x,y)}$ are zero [4], we obtain for the covariance $\mathbf{\Sigma}_p$, see [4],

$$\mathbf{\Sigma}_p = \sigma_t^2 \mathbf{\Gamma}_{\mathcal{R}}^{-1}(\mathbf{p}_a). \tag{9}$$

Expression (9) provides an inexpensive way to compute the reliability of the motion estimates. Although the matrix $\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_a)$ is in general unknown because it depends on the actual value $\mathbf{p}_a$ of the unknown vector $\mathbf{p}$, it can be approximated by $\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0)$ used in the iterative estimation algorithm. We note however that when the motion model is linear in the motion parameters, as it is the case with the majority of motion models used in practice, the matrix $\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p})$ becomes independent of the vector $\mathbf{p}$ because the derivatives of the displacement $\mathbf{d}(\mathbf{p})$ involved in (4) do not depend on the motion parameters. In this case, $\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0)$ does not change along the iterative estimation algorithm. The matrix $\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0)$ depends uniquely on the image region $\mathcal{R}$ and $\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0)$ will be denoted simply by $\mathbf{\Gamma}_{\mathcal{R}}$. Since the noise variance σ_t^2 is considered to be constant, we measure the error covariance for different regions by comparing the corresponding matrices $\mathbf{\Gamma}_{\mathcal{R}}^{-1}$. For example, the mean square Euclidean distance between the true vector $\mathbf{p}_a$ and the estimated vector $\widehat{\mathbf{p}}$, denoted by σ_p^2, is proportional to the trace of the matrix $\mathbf{\Gamma}_{\mathcal{R}}^{-1}$,

$$\sigma_p^2 = \mathrm{E}\left\{(\widehat{\mathbf{p}} - \mathbf{p}_a)^T(\widehat{\mathbf{p}} - \mathbf{p}_a)\right\} = \sigma_t^2 \,\mathrm{tr}\left(\mathbf{\Gamma}_{\mathcal{R}}^{-1}\right). \tag{10}$$

4. TRANSLATIONAL MOTION

The vector of motion parameters is defined as $\mathbf{p} = [p_1, p_2]^T$, determining the displacement as $\mathbf{d}(\mathbf{p}) = [p_1, p_2]^T = \mathbf{p}$.

Motion estimation To compute the matrix $\mathbf{\Gamma}_{\mathcal{R}}(\mathbf{p}_0)$ and the vector $\boldsymbol{\gamma}_{\mathcal{R}}(\mathbf{p}_0)$ needed for the Gauss-Newton iterates, we replace the gradient of $\mathbf{d}$ with respect to $\mathbf{p}$, $\nabla_{\mathbf{p}}\mathbf{d}^T = \mathbf{I}_{2\times 2}$, into (4) and (5), obtaining

$$\mathbf{\Gamma}_{\mathcal{R}} = \begin{bmatrix} \iint_{\mathcal{R}} i_x^2 \, dx\, dy & \iint_{\mathcal{R}} i_x i_y \, dx\, dy \\ \iint_{\mathcal{R}} i_x i_y \, dx\, dy & \iint_{\mathcal{R}} i_y^2 \, dx\, dy \end{bmatrix}, \qquad (11)$$

$$\boldsymbol{\gamma}_{\mathcal{R}}(\mathbf{p}_0) = -\begin{bmatrix} \iint_{\mathcal{R}} i_x i_t(\mathbf{p}_0) \, dx\, dy \\ \iint_{\mathcal{R}} i_y i_t(\mathbf{p}_0) \, dx\, dy \end{bmatrix}. \qquad (12)$$

The behavior of the estimation algorithm depends on the condition number of the matrix $\mathbf{\Gamma}_{\mathcal{R}}$ in (11). To see the influence of the brightness pattern within region $\mathcal{R}$ on the condition number $k(\mathbf{\Gamma}_{\mathcal{R}})$ consider that $\iint_{\mathcal{R}} i_x i_y \, dx\, dy = 0$. The matrix $\mathbf{\Gamma}_{\mathcal{R}}$ given by (11) becomes diagonal and the condition number given by (6) is simply

$$k(\mathbf{\Gamma}_{\mathcal{R}}) = \frac{\iint_{\mathcal{R}} i_x^2 \, dx\, dy}{\iint_{\mathcal{R}} i_y^2 \, dx\, dy} \quad \text{if} \quad \iint_{\mathcal{R}} i_x^2 \, dx\, dy \geq \iint_{\mathcal{R}} i_y^2 \, dx\, dy$$

or the inverse if the inequality goes in the opposite way. If one of the components of the spatial image gradient is much larger than the other, $k(\mathbf{\Gamma}_{\mathcal{R}})$ becomes large and the equation system (3) is ill-conditioned. The condition $k(\mathbf{\Gamma}_{\mathcal{R}}) < \lambda$, where λ is a threshold, restricts the brightness pattern within region $\mathcal{R}$ not to have variability along some direction much higher than the variability along the perpendicular direction.

The analysis in the paragraph above explains the well known *aperture problem*. The aperture problem is usually described as the impossibility of estimating locally the 2D motion. In fact, if the region $\mathcal{R}$ contains a single pixel, the matrix $\mathbf{\Gamma}_{\mathcal{R}}$ given by (11) is singular; we obtain $\det(\mathbf{\Gamma}_{\mathcal{R}}) = 0$ by removing the integrals from (11), and the condition number $k(\mathbf{\Gamma}_{\mathcal{R}})$ is $+\infty$. This happens because the two motion parameters can not be determined by the single constraint imposed by the brightness constancy. The study of the condition number $k(\mathbf{\Gamma}_{\mathcal{R}})$ shows that, for particular structures of the brightness pattern, it is very difficult to estimate the 2D motion, even when the region $\mathcal{R}$ contains several pixels.

Although the previous analysis was obtained with $\iint_{\mathcal{R}} i_x i_y \, dx\, dy = 0$, we should note that the case $\iint_{\mathcal{R}} i_x i_y \, dx\, dy \neq 0$ does not correspond to a more general situation. In fact, it can be shown that an appropriate rotation of the brightness pattern makes $\iint_{\mathcal{R}} i_x i_y \, dx dy = 0$, without changing the condition number $k(\mathbf{\Gamma}_{\mathcal{R}})$ – as we would expect, the conditioning of the estimation of the motion of the brightness pattern is independent of 2D rigid transformations of the brightness pattern.

Figure 1 illustrates the dependence of the conditioning of the 2D motion estimation on the structure of the brightness pattern. For each of the eight 10×10 images in Figure 1, we determine the condition number of the matrix $\mathbf{\Gamma}_{\mathcal{R}}$. The condition number $k(\mathbf{\Gamma}_{\mathcal{R}})$, obtained by evaluating expression (6), is on the bottom of each image in Figure 1. The texture of the brightness pattern shown on the top left image is such that there is no dominant direction over the entire region $\mathcal{R}$. We expect that the estimation of the 2D motion of a pattern of this kind is very well conditioned. In fact, over the entire image no component of the spatial gradient dominates, and the condition number $k(\mathbf{\Gamma}_{\mathcal{R}})$ captures this behavior. We get $k(\mathbf{\Gamma}_{\mathcal{R}}) = 1.34$ – the value of $k(\mathbf{\Gamma}_{\mathcal{R}})$ is close to unity indicating that the linear system involved in the Gauss-Newton iterates of the motion estimation algorithm is well conditioned. In contrast to this case, the texture of the brightness pattern shown in the bottom right image of Figure 1 exhibits a clear dominant direction. It is very hard to perceive the 2D motion of these type of patterns because only the component of the motion that is perpendicular to the dominant direction of the texture is perceived. The condition number $k(\mathbf{\Gamma}_{\mathcal{R}})$ captures the indetermination in estimating the 2D motion – it is $k(\mathbf{\Gamma}_{\mathcal{R}}) = 133.82$ indicating that the linear system involved in the Gauss-Newton iterates of the motion estimation algorithm is ill-conditioned. The other images of Figure 1 illustrate intermediate cases.

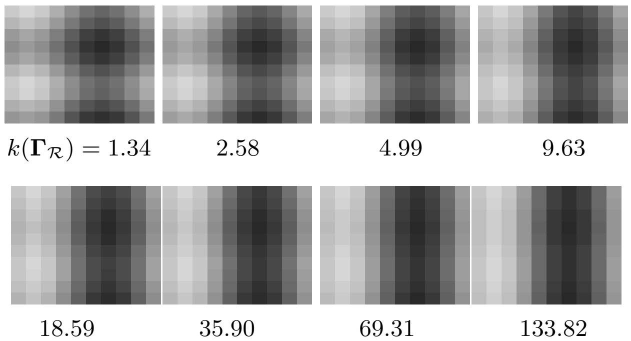

Fig. 1. When the texture of the brightness pattern exhibits a dominant direction, the motion estimation is ill conditioned – see the bottom right image and the high value of the condition number of the matrix involved in the algorithm.

Estimation error The covariance matrix of the estimation error for the translational motion model is given by (9) after replacing $\mathbf{\Gamma}_{\mathcal{R}}$ by expression (11). The mean square error of the displacement estimate is the trace of the covariance matrix $\mathbf{\Sigma}_p$, see (10). In terms of image gradients, we get the following expression for the mean square error, denoted by σ_p^2,

$$\sigma_p^2 = \sigma_t^2 \frac{\int_{\mathcal{R}} i_y^2 \, dx\, dy + \int_{\mathcal{R}} i_x^2 \, dx\, dy}{\int_{\mathcal{R}} i_x^2 \, dx\, dy \, \int_{\mathcal{R}} i_y^2 \, dx\, dy - \left(\int_{\mathcal{R}} i_x i_y \, dx\, dy\right)^2}. \qquad (14)$$

In [2, 4], when recovering 3D structure from 2D motion estimates, we use the estimate of the mean square error σ_p^2 given by expression (14) to improve the 3D recontruction by weighting differently motion estimates corresponding to different regions of the image.

To interpret the mean square error σ_p^2 given by (14), let us consider again that the matrix $\mathbf{\Gamma}_{\mathcal{R}}$ is diagonal. This is the general case because, as for the condition number, it can be shown that any non-diagonal matrix $\mathbf{\Gamma}_{\mathcal{R}}$ can be made diagonal without changing σ_p^2, by an appropriate rotation of the image brightness pattern. When $\iint_{\mathcal{R}} i_x i_y \, dx \, dy = 0$, the mean square error σ_p^2 is

$$\sigma_p^2 = \sigma_t^2 \left[\left(\iint_{\mathcal{R}} i_x^2 \, dx \, dy \right)^{-1} + \left(\iint_{\mathcal{R}} i_y^2 \, dx \, dy \right)^{-1} \right],$$

showing that the error in estimating the displacement is proportional to the inverse of the sum of the square components of the image gradient within region $\mathcal{R}$. This coincides with the intuitive notion that the higher the spatial variability of the brightness pattern is, the lower the error in estimating the motion is. As expected, it is also clear that the estimation error decreases when the size of the region $\mathcal{R}$ increases.

Figure 2 illustrates the dependence of the expected square error of the 2D motion estimates on the image brightness pattern. To isolate the estimation error from the eventual ill-posedness of the motion estimation problem, we used brightness patterns that do not have a dominant texture direction, i.e., we used brightness patterns for which the linear system involved in the motion estimation is well conditioned. In particular, we used the brightness pattern of the top left image of Figure 1 to generate all the images of Figure 2 by changing the brightness contrast. The conditioning of the linear system involved in the motion estimation problem does not depend on the brightness contrast, as shown by the constant value of the condition number, $k(\mathbf{\Gamma}_{\mathcal{R}}) = 1.34$ for all the images in Figure 2.

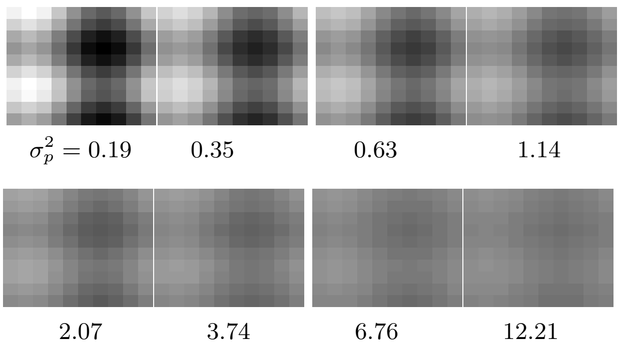

Fig. 2. The dependence of the motion estimation error on the image brightness pattern. The expected square error σ_p^2 increases with the decrease of the brightness contrast.

For each image in Figure 2, we computed the mean square error σ_p^2 by evaluating expression (14). Since the goal is to illustrate the influence of the brightness pattern on σ_p^2, we made $\sigma_t^2 = 1$ when evaluating expression (14). The values obtained for σ_p^2 are shown in Figure 2 on the bottom of the corresponding image. The top left image of Figure 2 has a very high brightness contrast. For this reason, we expect that the estimate of the 2D motion of such a pattern is very accurate. In fact, the sum of the square components of the image gradient has a high value and the value of the motion estimation mean square error is low, $\sigma_p^2 = 0.19$. When the brightness contrast decreases, we expect less accurate motion estimates. The values of σ_p^2 in Figure 2 are in agreement with this. The expected square error σ_p^2 increases with the decrease of brightness contrast because the square components of the image gradient decrease. The bottom right image of Figure 2 shows the extreme situation of a pattern with almost zero brightness contrast. For this pattern, the expected mean square estimation error is very high – larger than 60 times the error for the top left image. It is therefore hopeless to try to compute accurate motion estimates for this kind of low contrast patterns. Note that this is due to the fundamental bound on the motion estimation error, not to the conditioning of the linear system involved in the motion estimation algorithm (the condition number $k(\mathbf{\Gamma}_{\mathcal{R}}) = 1.34$ is close to unity indicating that the linear system is well conditioned).

5. CONCLUSION

We discussed the conditioning of the 2D motion estimation problem and derived expressions for the covariance of the estimation error for general parametric motion models. We specialized this analysis to the translational motion model. For this model, we related the convergence of the estimation algorithm to the variability of the brightness pattern – for the algorithm to be stable, the two components of the image gradient should not have too radically different magnitude values – and derive the expression for the expected square of the Euclidean distance between the true and estimated values of the parameters – when the magnitude of the components of the image gradient is large, the error is smaller.

6. REFERENCES

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