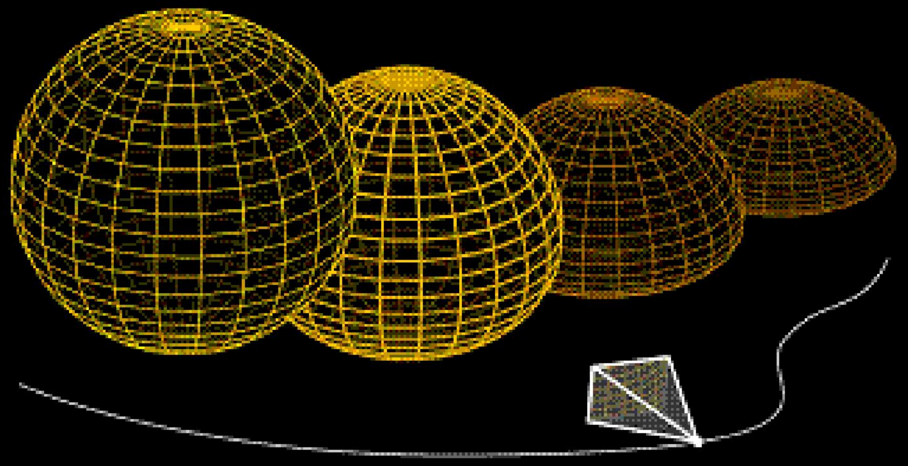
Adaptive metric registration of 3D Models to Non-rigid image trajectories



Alessio Del Bue

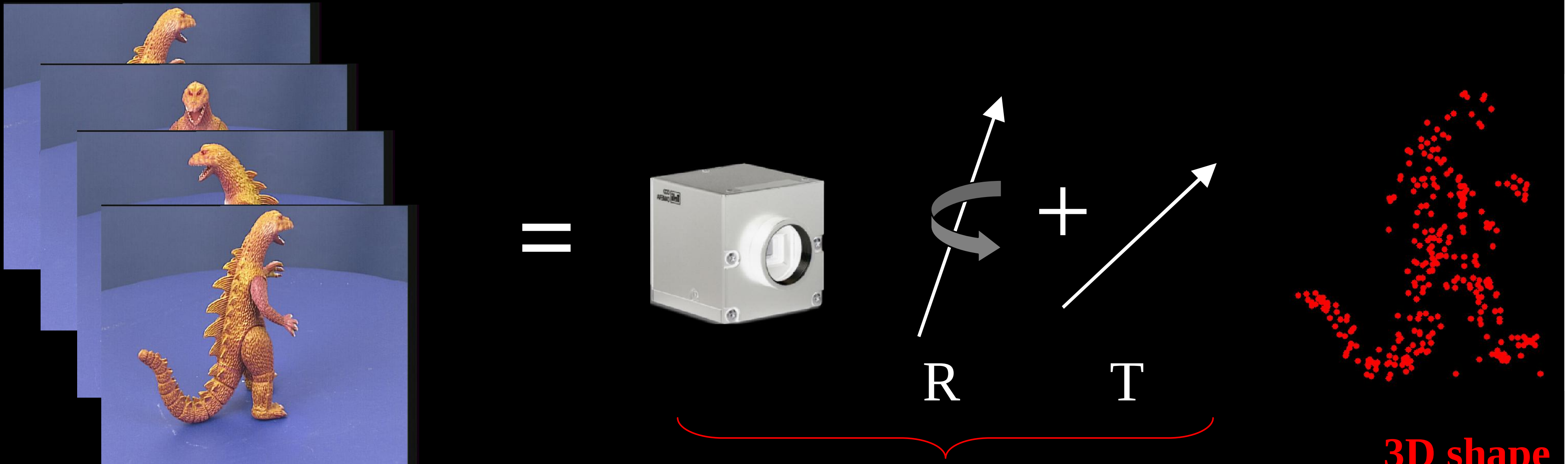
PLUS LAB

Italian Institute of Technology

ECCV 2010, Crete, 7th of September, 2010

Image Sequence Registration

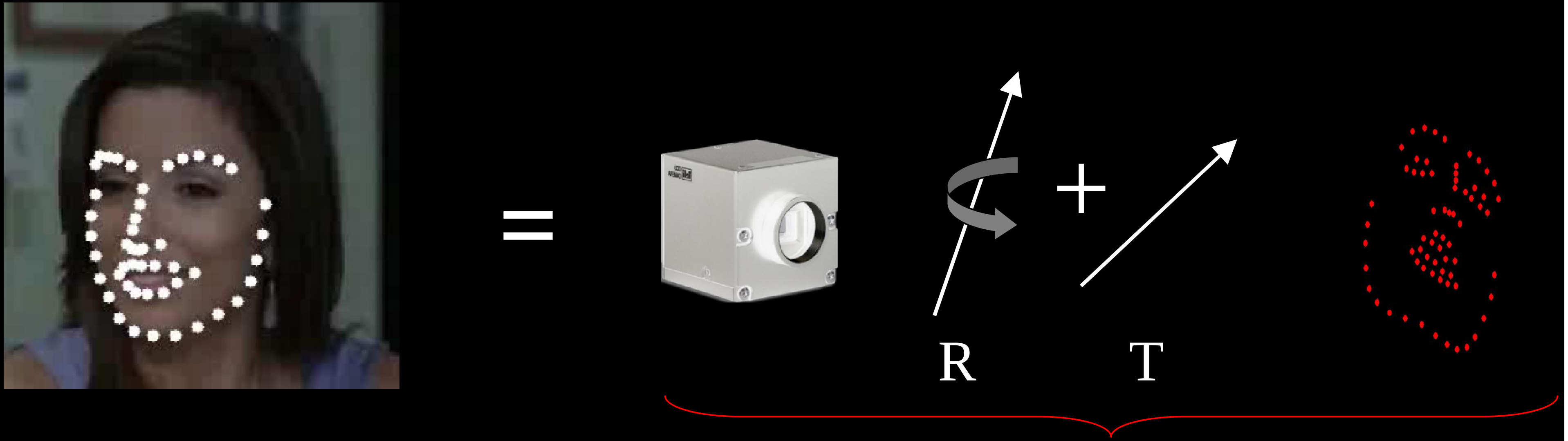
Rigid



Unknowns

3D shape

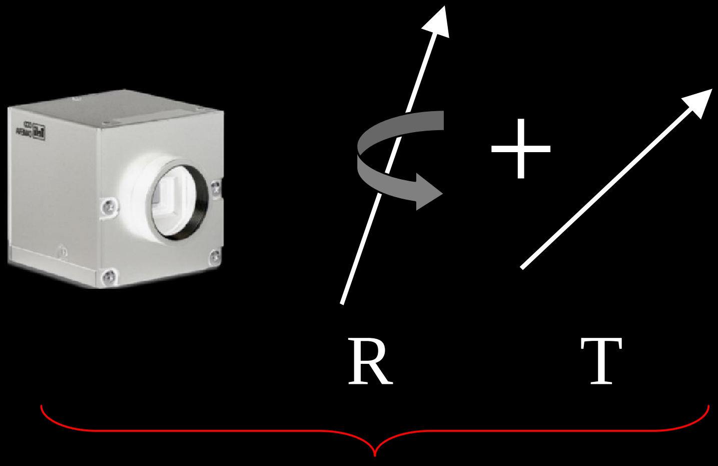
Adaptive



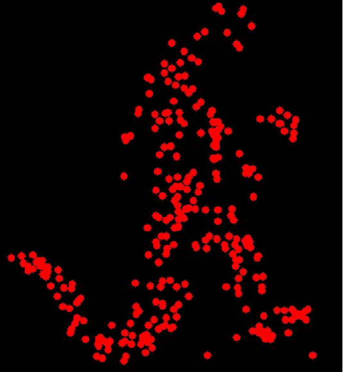
Unknowns

Rigid Registration

$$\begin{bmatrix} u_{11} & \cdots & u_{1P} \\ v_{11} & \cdots & v_{1P} \\ u_{21} & \cdots & u_{2P} \\ v_{21} & \cdots & v_{2P} \\ u_{51} & \cdots & u_{5P} \\ v_{51} & \cdots & v_{5P} \\ u_{41} & \cdots & u_{4P} \\ v_{41} & \cdots & v_{4P} \end{bmatrix} =$$



Unknowns

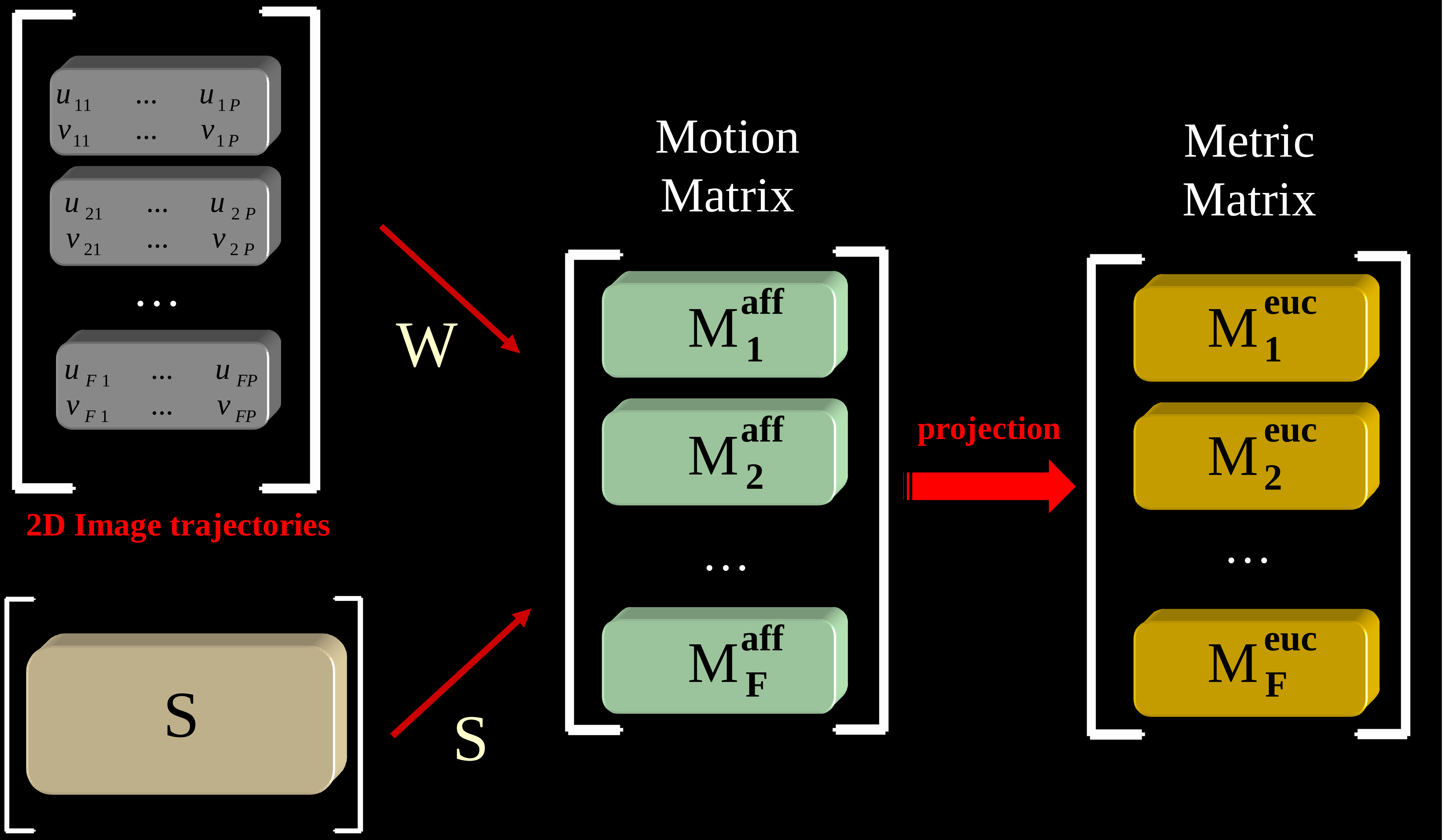


3D shape

Rigid Registration

$$\begin{bmatrix} u_{11} & \cdots & u_{1P} \\ v_{11} & \cdots & v_{1P} \\ u_{21} & \cdots & u_{2P} \\ v_{21} & \cdots & v_{2P} \\ u_{51} & \cdots & u_{5P} \\ v_{51} & \cdots & v_{5P} \\ u_{41} & \cdots & u_{4P} \\ v_{41} & \cdots & v_{4P} \end{bmatrix} = \begin{bmatrix} \mathrm{M} \end{bmatrix} \begin{bmatrix} \mathrm{S} \end{bmatrix}$$

1. Rigid Registration - Affine

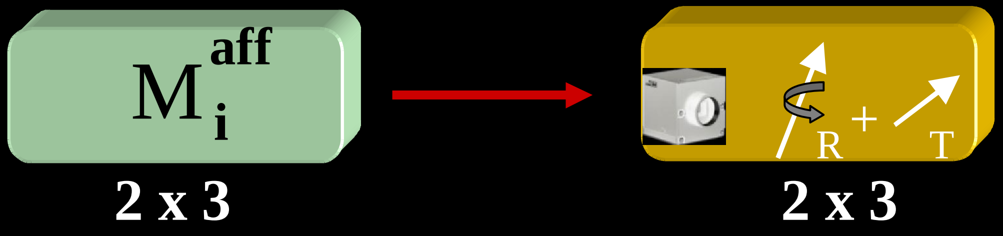


2D Image trajectories

3D shape (known)

2. Rigid Registration – Metric

Problem: Given an initial solution find the matrix which respects the given metric constraints.



- The constraints on N_i depends on the given camera model, in this case we consider the simplest **orthographic camera** model:

$$\underset{2 \times 3}{M_i^{euc}}\ \underset{3 \times 2}{M_i^{euc}}{}^{T} = \underset{2 \times 2}{\begin{matrix}1 & 0\\ 0 & 1\end{matrix}}$$

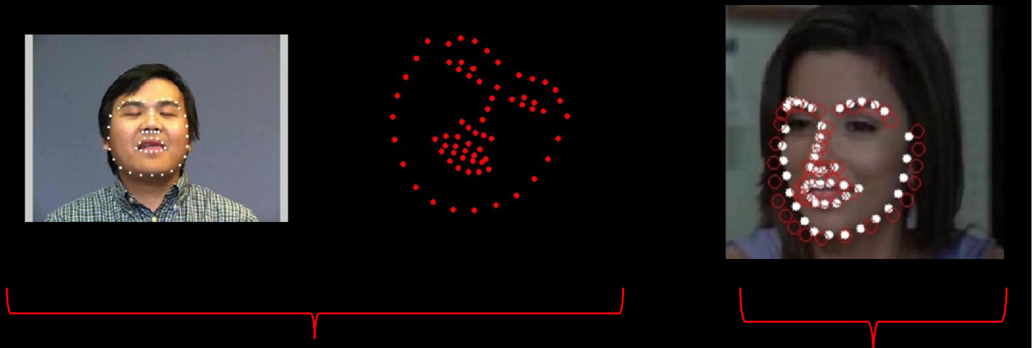
- The projection to the closest orthographic camera matrix can be solved optimally via SVD

$$\underset{2 \times 3}{M_i^{aff}} \xrightarrow[\text{(econ)}]{\text{SVD}} \underset{2 \times 2}{U}\ \underset{2 \times 2}{D}\ \underset{2 \times 3}{V^T}$$

$$\underset{2 \times 3}{M_i^{euc}} = \underset{2 \times 2}{U}\ \underset{2 \times 3}{V^T}$$

Adaptive Metric Registration

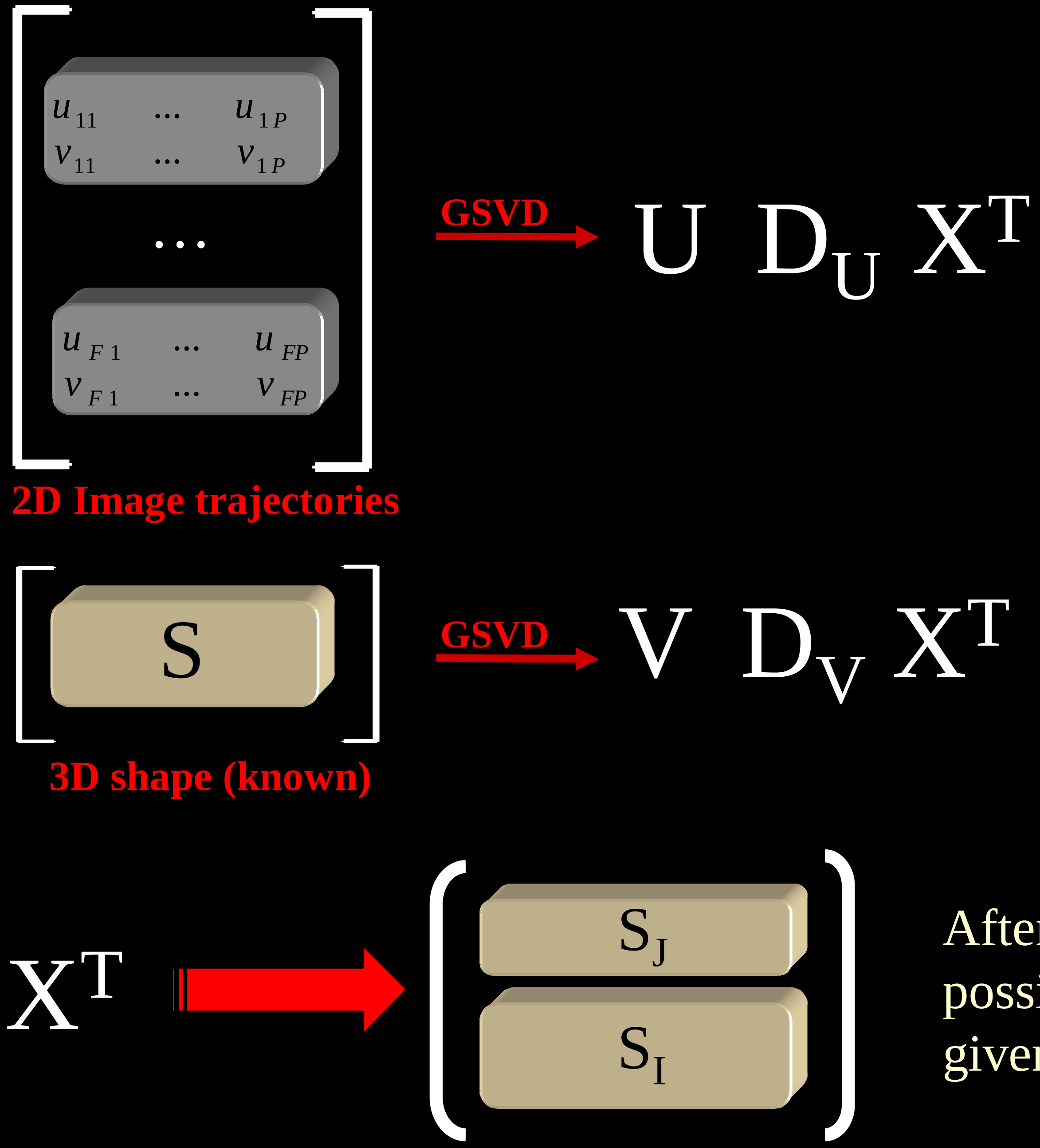
Problem: Now the 3D shape is not an exact description of the observed 2D image trajectories. Thus we need to estimate a new shape that adapts to the measured 2D data.



Proposed Solution: A two step closed form procedure.

1. Find a common bilinear decomposition for the 2D image trajectories and the shape to register
2. Find the best transformation which enforces the metric constraints for both the 2D trajectories and the 3D shape to register

1. *Find a joint decomposition*



$$\begin{bmatrix} \begin{matrix} u_{11} & \dots & u_{1P} \\ v_{11} & \dots & v_{1P} \end{matrix} \\ \dots \\ \begin{matrix} u_{F1} & \dots & u_{FP} \\ v_{F1} & \dots & v_{FP} \end{matrix} \end{bmatrix} \xrightarrow{\text{GSVD}} U \; D_U \; X^T$$

2D Image trajectories

$$\begin{bmatrix} S \end{bmatrix} \xrightarrow{\text{GSVD}} V \; D_V \; X^T$$

3D shape (known)

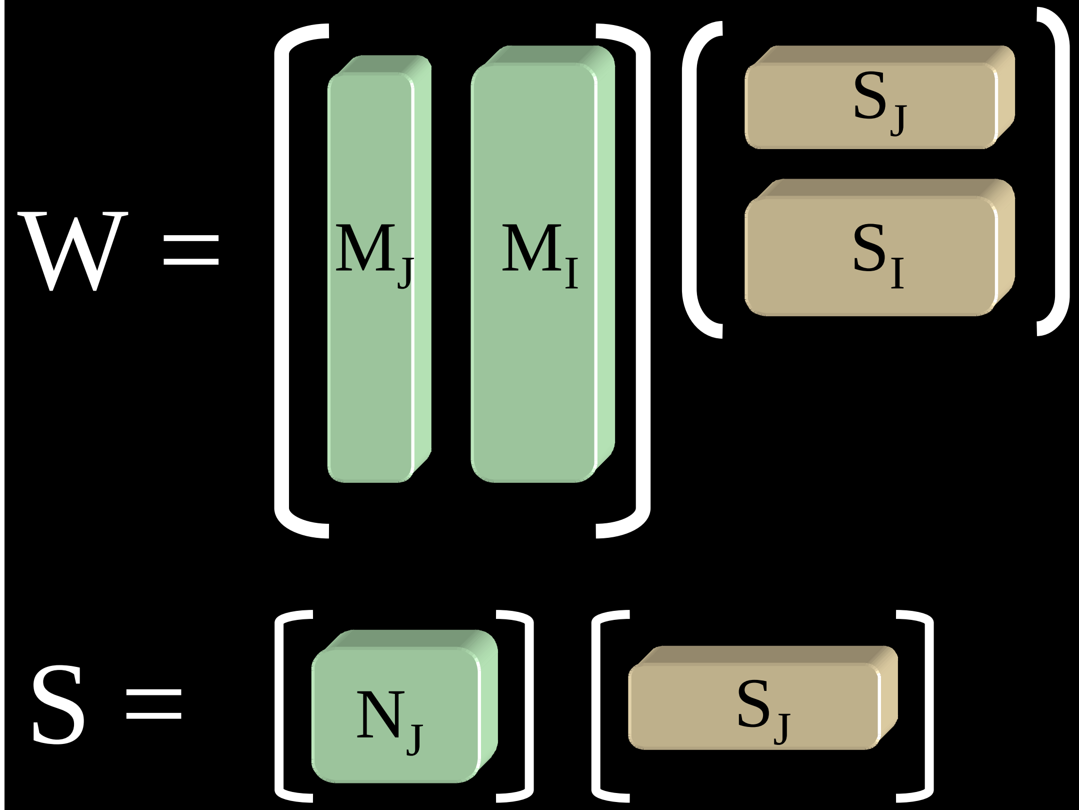
$$X^T \Rightarrow \begin{pmatrix} S_J \\ S_I \end{pmatrix}$$

- Common row space X^T given the image trajectory and 3D shape
- Generalised singular values such that: $D_u^2 + D_V^2 = I$
- Orthogonal column space such that $U\,U^T = I$ and $V\,V^T = I$

After simple matrix manipulations it is possible to extract an affine joint shape given both data.

A. Del Bue, "A factorization approach to structure from motion with shape priors," CVPR, 2008.

2. Adaptive Registration – Metric



$$W = \begin{bmatrix} M_J & M_I \end{bmatrix} \begin{pmatrix} S_J \\ S_I \end{pmatrix}$$

$$S = \begin{bmatrix} N_J \end{bmatrix} \begin{bmatrix} S_J \end{bmatrix}$$

- The solution from GSVD is affine, force the metric constraints given the camera matrix.
- There is an infinite set of solutions for the joint factorisation:

$$\mathbf{W_j = M_j\,S_j} \quad \text{and} \quad \mathbf{S = N_j\,S_j}$$

- Since any full rank matrix $Q_{3\ x\ 3}$ gives:

$$\mathbf{W_j = M_j\,Q\,Q^{-1}\,S_j} \quad \text{and} \quad \mathbf{S = N_j\,Q\,Q^{-1}\,S_j}$$

$$\mathbf{W_j = \tilde{M}_j\,\tilde{S}_j} \quad \text{and} \quad \mathbf{S = \tilde{N}_j\,\tilde{S}_j}$$

- Find the corrective transform Q that enforces the metric constraints both for the 2D trajectories and the known 3D shape.
- We propose a new set of constraints given the 3D metric constraints in S i.e. N_j should represent a rotation.

2. Joint Metric Constraints

$$\underset{2 \text{ x } 3}{M_i}\;\underset{3 \text{ x } 2}{M_i^{T}} = \underset{2 \text{ x } 2}{\begin{matrix} a & c \\ c & b \end{matrix}} \qquad\qquad \underset{3 \text{ x } 3}{N_J}\;\underset{3 \text{ x } 3}{N_J^{T}} = \underset{3 \text{ x } 3}{C}$$

and

$$\underset{2 \text{ x } 3}{M^{euc}_i}\;\underset{3 \text{ x } 2}{M^{euc\,T}_i} = \underset{2 \text{ x } 2}{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}} \qquad\qquad \underset{3 \text{ x } 3}{N^{euc}_J}\;\underset{3 \text{ x } 3}{N^{euc\,T}_J} = \underset{3 \text{ x } 3}{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}$$

Find the corrective transform Q such that:

$$\underset{2 \text{ x } 3}{M_i}\;\underset{3 \text{ x } 3}{Q}\;\underset{3 \text{ x } 3}{Q^{T}}\;\underset{3 \text{ x } 2}{M_i^{T}} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

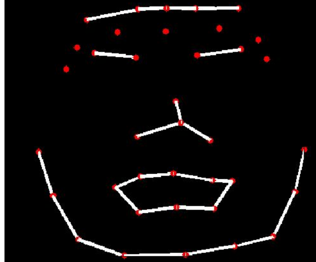
$$\underset{3 \text{ x } 3}{N_J}\;\underset{3 \text{ x } 3}{Q}\;\underset{3 \text{ x } 3}{Q^{T}}\;\underset{3 \text{ x } 3}{N_J^{T}} = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Two solutions for Q are proposed: one with Least Squares and the other with Convex optimisation (in the paper).

Experiments

Synthetic Experiments

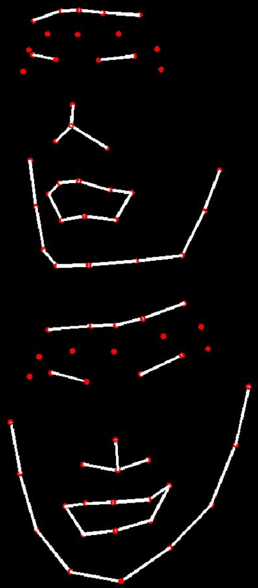
Motion capture
ground truth

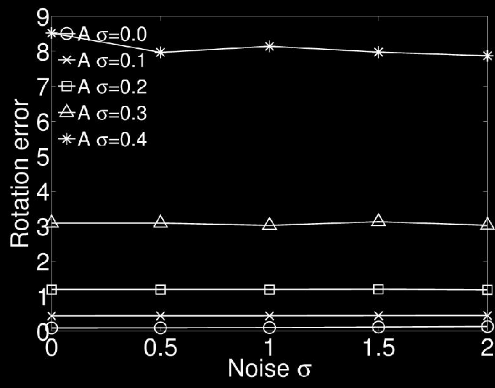


3D shape

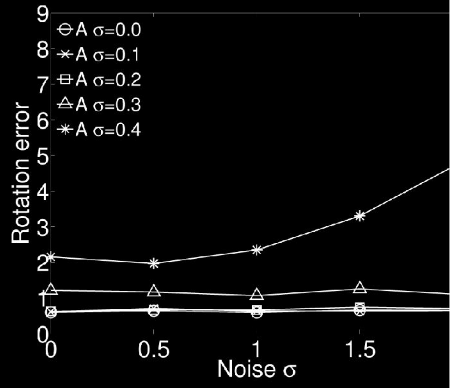
Apply

Distortion



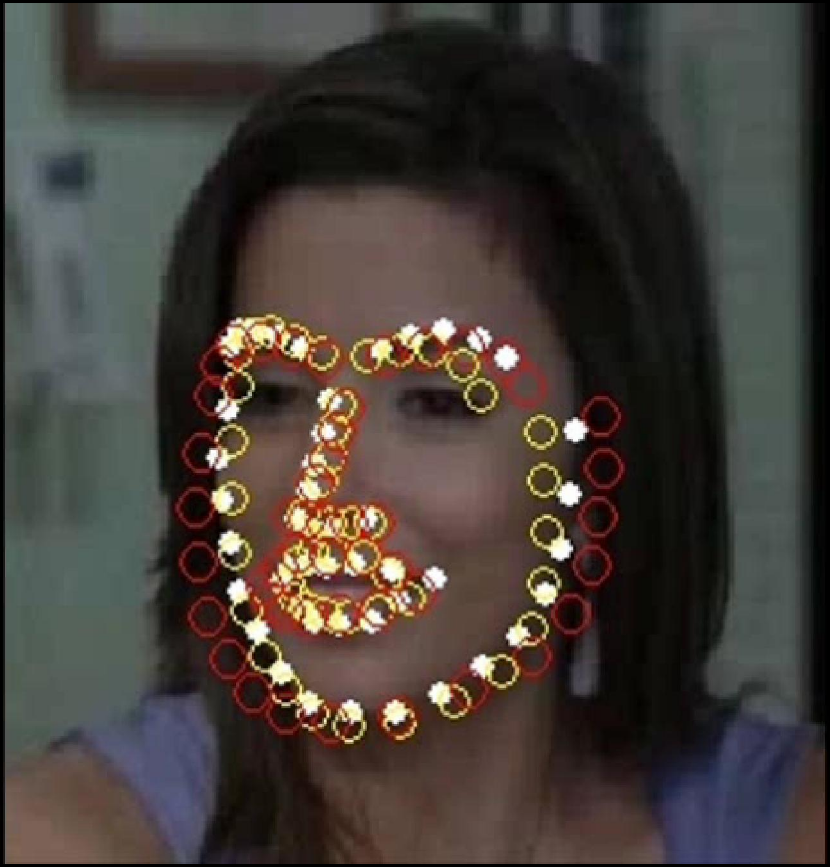


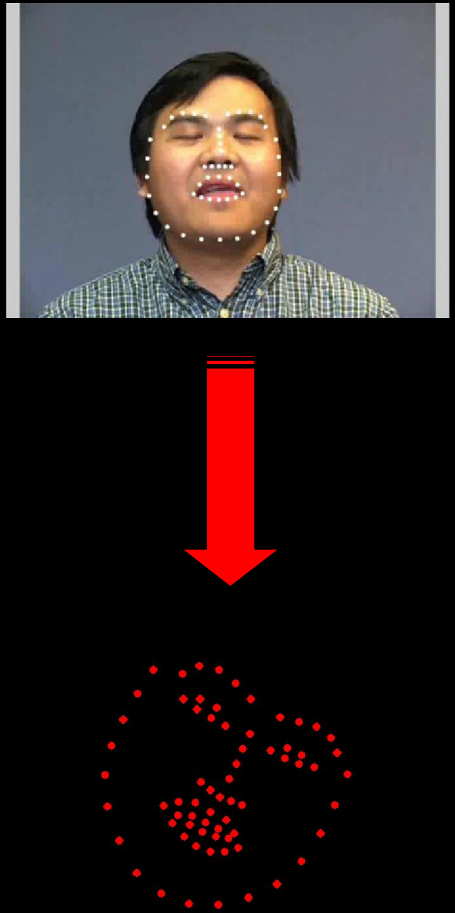
Rigid



Adaptive

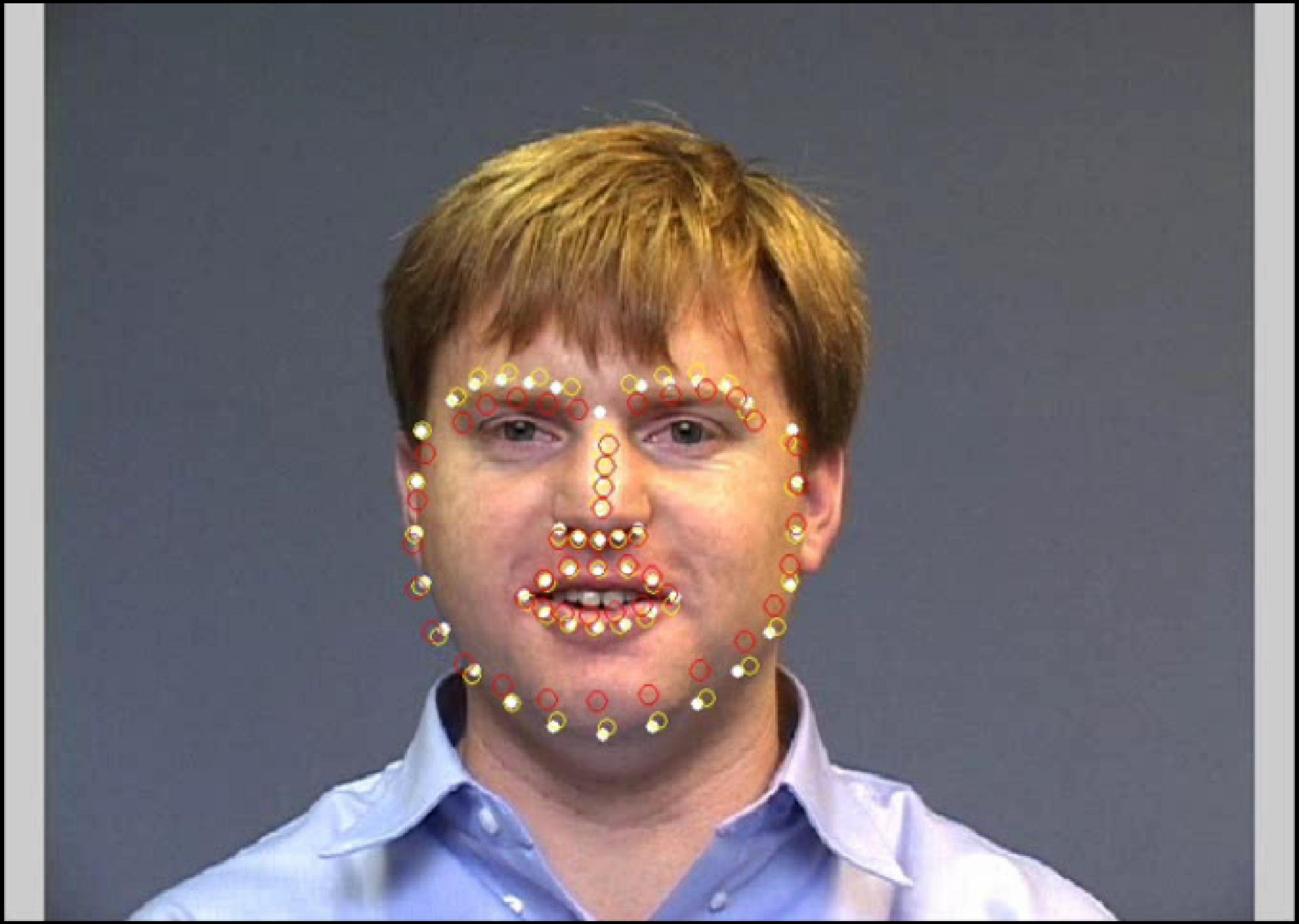
Real Experiments





- White dots represent the tracked 2D data (Tracks and images courtesy of Julien Peyras)
- ○ Red circles show the registration without adaptation.
- ○ Yellow circles show the registration with adaptation.

Real Experiments

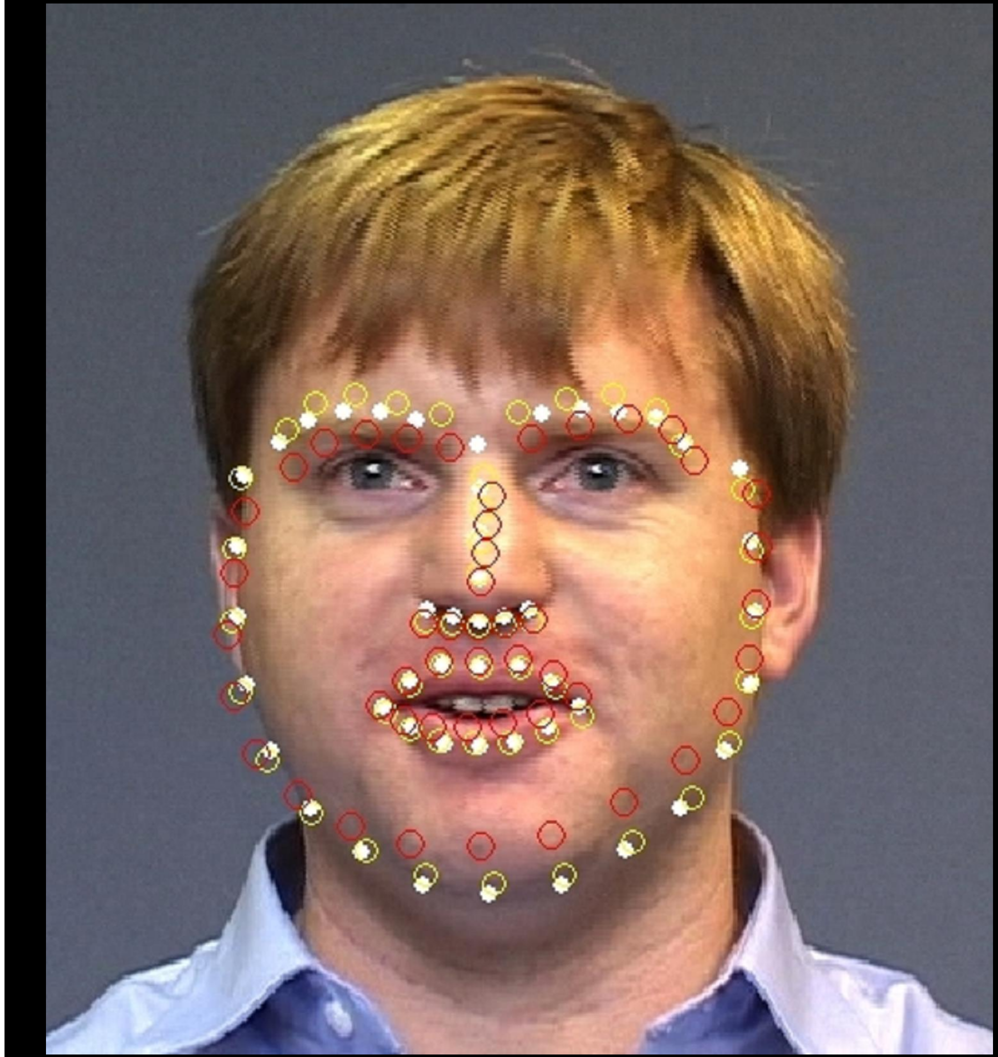


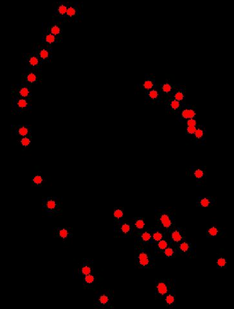
- White dots represent the tracked 2D data (Tracks and images courtesy of Jing Xiao)

○ Red circles show the registration without adaptation.

○ Yellow circles show the registration with adaptation.

Real Experiments

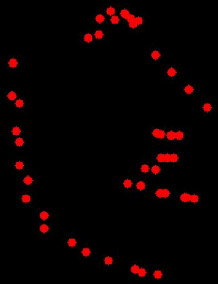




Shape without joint metric constraints

$$R_i R_i^T = I$$

$$\cancel{Z_J Z_J^T = I}$$



Shape using joint metric constraints

$$R_i R_i^T = I$$

$$Z_J Z_J^T = I$$

- White dots represent the tracked 2D data (Tracks and images courtesy of Jing Xiao)

○ Red circles show the registration without adaptation.

○ Yellow circles show the registration with adaptation.

Real Experiments – Missing Data

Missing data algorithm:

0. Find an initialization for the missing entries in the measurement matrix W

1. Compute the joint image registration and find M_j and S_j.

2. Given this solution, inpute the missing entries as $W = M_j\ S_j$.

3. Go back to 1. until convergence is reached.

- White dots represent the tracked 2D data (Tracks and images courtesy of Julien Peyras)

○ Yellow circles show the registration with adaptation.

Conclusions

To summarise:

Adaptive metric registration is made by a two step closed-form procedure:

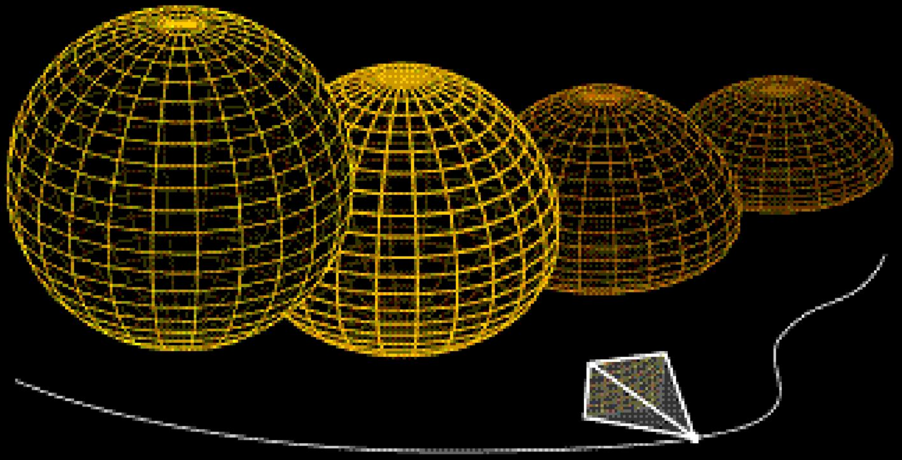
1. Compute a common bilinear decomposition with GSVD for the 2D image trajectories and the shape to register
2. Find the best transformation which enforces the metric constraints for both the 2D trajectories and the 3D shape to register

Further work:

The proposed framework can be extended to any data that can be expressed in bilinear form i.e. photometric stereo, non-rigid and articulated SfM.

Extend the method to deal with outliers – when the 3D shape and 2D data have mismatches GSVD may not find a common decomposition.

Adaptive metric registration of 3D Models to Non-rigid image trajectories



For more information:
www.isr.ist.utl.pt/~adb/