Nonlinear Control Systems

Homework #4

(Due date: March 28, 2012)

March 23, 2012

1. For a linear system

$$\dot{x} = Ax + Bu,$$

the state-feedback control law that minimizes the quadratic cost function

$$J = \int_0^\infty \left(x^T Q x + u^T R u\right) dt, \quad R = R^T > 0, \;\; Q = Q^T \geq 0$$

is given by

$$u = -Kx,$$

where $K = R^{-1}B^T P$ and $P = P^T > 0$ satisfies the Riccati equation

$$PA + A^T P + Q - PBR^{-1}B^T P = 0.$$

Using $V(x) = x^T P x$ as Lyapunov function candidate, show that the origin is globally asymptotically stable (GAS) when

a) $Q > 0$

b) $Q = C^T C$ and the pair (A, C) is observable.

Hint: Recall that for an observable pair (A, C), the vector $Ce^{At}x = 0$, $\forall t \geq 0$, if and only if $x = 0$.