

# Nonlinear Control Systems

## Homework #2

(Due date: March 14, 2012)

March 8, 2012

1. Let  $f, g : D \subset \mathbb{R}^n \rightarrow \mathbb{R}$  be  $C^1$  functions, such that

$$|f(x) - g(x)| < \epsilon, \quad \forall x \in D$$

Consider the following dynamical systems

$$\dot{x} = f(x), \quad x(0) = a$$

$$\dot{y} = g(y), \quad y(0) = a$$

Show that

$$\|x(t) - y(t)\| \leq \frac{\epsilon}{L}(e^{Lt} - 1),$$

where  $L$  is the Lipschitz constant, i.e.,

$$\|f(x) - f(y)\| \leq L\|x - y\|, \quad \forall x, y \in D$$

Tip: Use Gronwall's inequality.