KINEMATICS MODELS OF MOBILE ROBOTS

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References

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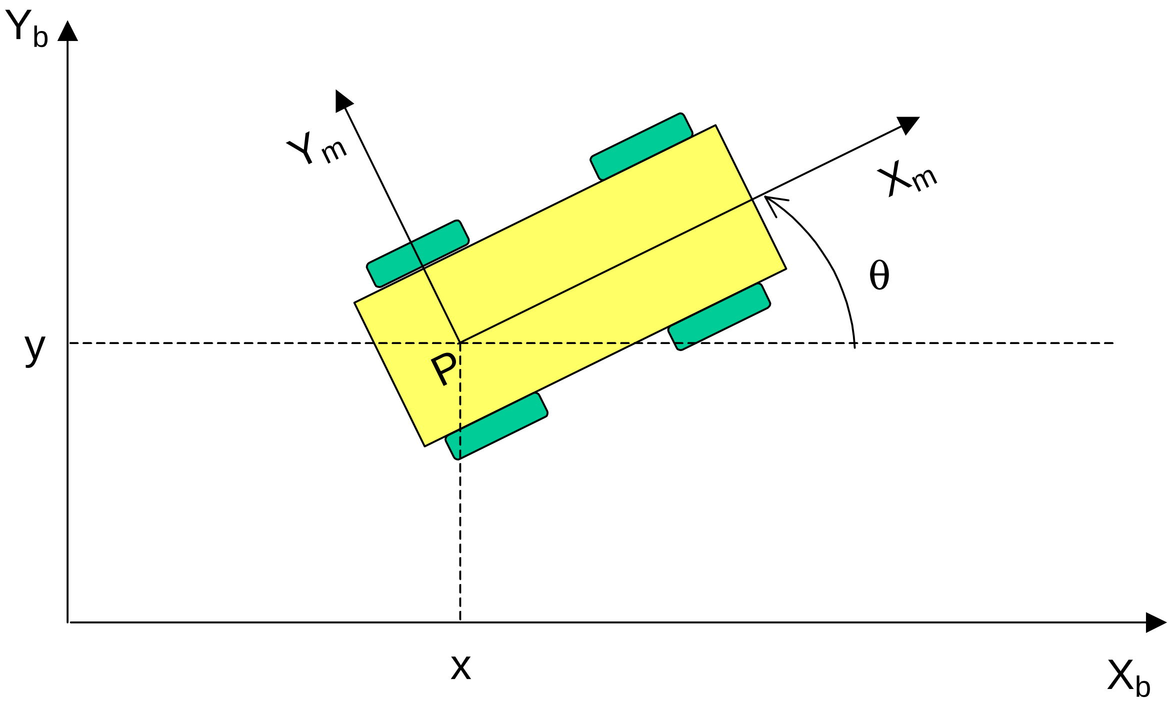
- Carlos Canudas de Wit, Bruno Siciliano, Georges Bastin (eds), “Theory of Robot Control”, Springer 1996.

Kinematics for Mobile Robots

- What is a **kinematic** model ?
- What is a **dynamic** model ?
- Which is the difference between kinematics and dynamics?

- **Locomotion** is the process of causing an autonomous robot to move.
  - In order to produce motion, forces must be applied to the vehicle
- **Dynamics** – the study of motion in which these forces are modeled
  - Includes the energies and speeds associated with these motions
- **Kinematics** – study of the mathematics of motion withouth considering the forces that affect the motion.
  - Deals with the geometric relationships that govern the system
  - Deals with the relationship between control parameters and the beahvior of a system in state space.

Notation



- $\{X_m, Y_m\}$ – moving frame
- $\{X_b, Y_b\}$ – base frame

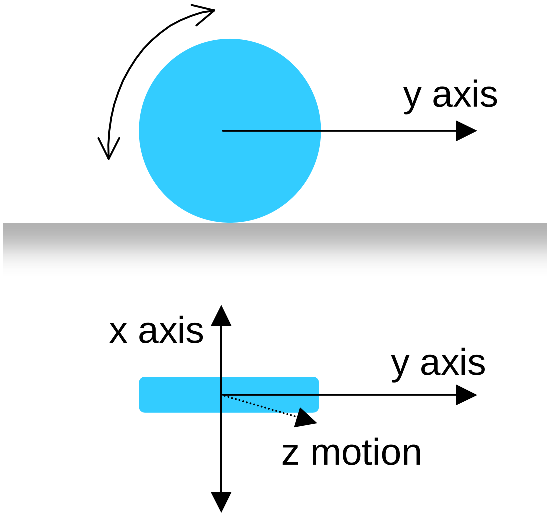
$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

robot posture in base frame

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation matrix expressing the orientation of the base frame with respect to the moving frame

- Idealized rolling wheel



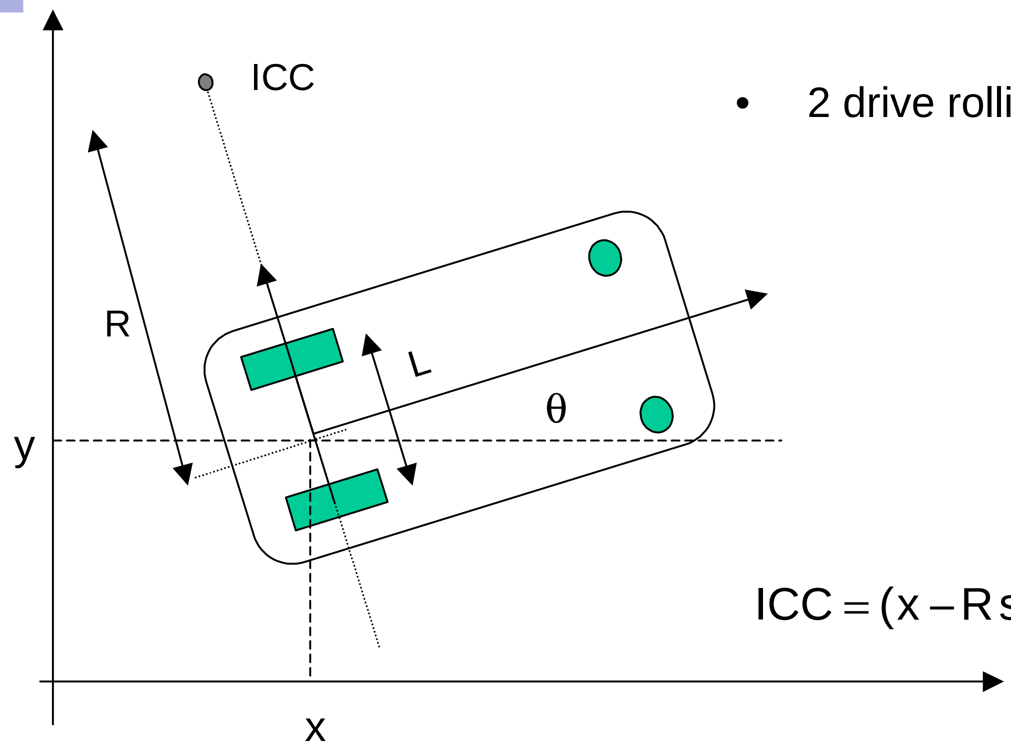
- If the wheel is free to rotate about its axis (x axis), the robot exhibits preferencial rollong motion in one direction (y axis) and a certain amount of lateral slip.

- For low velocities, rolling is a reasonable wheel model.
  – This is the model that will be considered in the kinematics models of WMR

Wheel parameters:
- r = wheel radius
- v = wheel linear velocity
- w = wheel angular velocity

Differential Drive

- 2 drive rolling wheels



$$ICC = (x - R\sin\theta,\ y + R\cos\theta)$$

- $v_r(t)$ – linear velocity of right wheel
- $v_l(t)$ – linear velocity of left wheel
- r – nominal radius of each wheel
- R – instantaneous curvature radius of the robot trajectory, relative to the mid-point axis

control variables

$R - \frac{L}{2}$ —— Curvature radius of trajectory described by LEFT WHEEL

$R + \frac{L}{2}$ —— Curvature radius of trajectory described by RIGHT WHEEL

$$w(t) = \frac{v_r(t)}{R + L/2}$$

$$w(t) = \frac{v_l(t)}{R - L/2}$$

$$w(t) = \frac{v_r(t) - v_l(t)}{L}$$

$$R = \frac{L}{2}\frac{(v_l(t) + v_r(t))}{(v_l(t) - v_r(t))}$$

$$v(t) = w(t)R = \frac{1}{2}(v_r(t) + v_l(t))$$

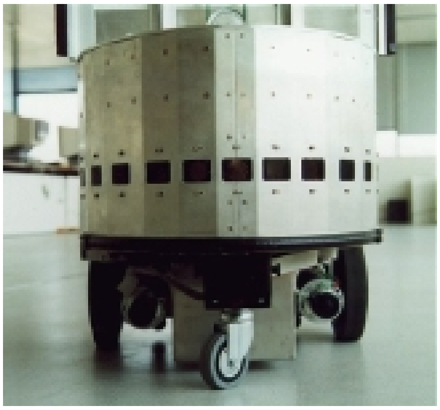
Differential Drive

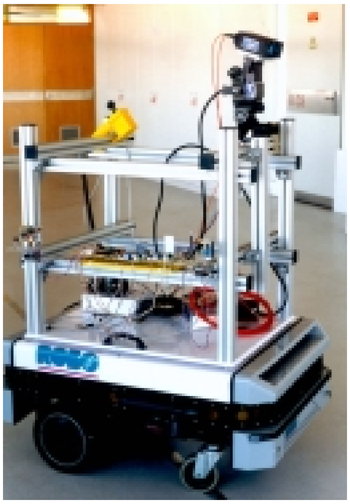
Kinematic model in the robot frame

$$\begin{bmatrix} v_x(t) \\ v_y(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ -r/L & r/L \end{bmatrix} \begin{bmatrix} w_l(t) \\ w_r(t) \end{bmatrix}$$

- $w_r(t)$ – angular velocity of right wheel
- $w_l(t)$ – angular velocity of left wheel

Useful for velocity control





Kinematic model in the world frame

$$v(t) = w(t)R = \frac{1}{2}(v_r(t) + v_l(t))$$

$$w(t) = \frac{v_r(t) - v_l(t)}{L}$$

$$\begin{aligned} \dot{x}(t) &= v(t)\cos\theta(t) \\ \dot{y}(t) &= v(t)\sin\theta(t) \\ \dot{\theta}(t) &= w(t) \end{aligned} \iff \begin{aligned} x(t) &= \int_0^t v(\sigma)\cos(\theta(\sigma))d\sigma \\ y(t) &= \int_0^t v(\sigma)\sin(\theta(\sigma))d\sigma \\ \theta(t) &= \int_0^t w(\sigma)d\sigma \end{aligned}$$

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix}$$

$$\dot{q}(t) = S(q)\xi(t)$$

control variables

- Particular cases:

  - $v_l(t)=v_r(t)$

    - **Straight line trajectory**

$$v_r(t) = v_l(t) = v(t)$$
$$w(t) = 0 \quad \Rightarrow \quad \dot{\theta}(t) = 0 \quad \Rightarrow \quad \theta(t) = \text{cte.}$$

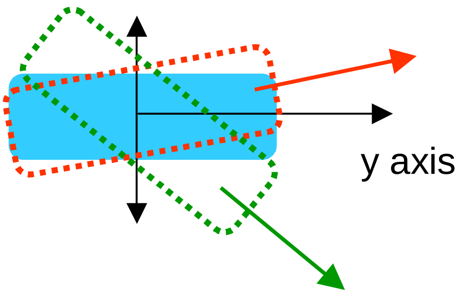
  - $v_l(t)=-v_r(t)$

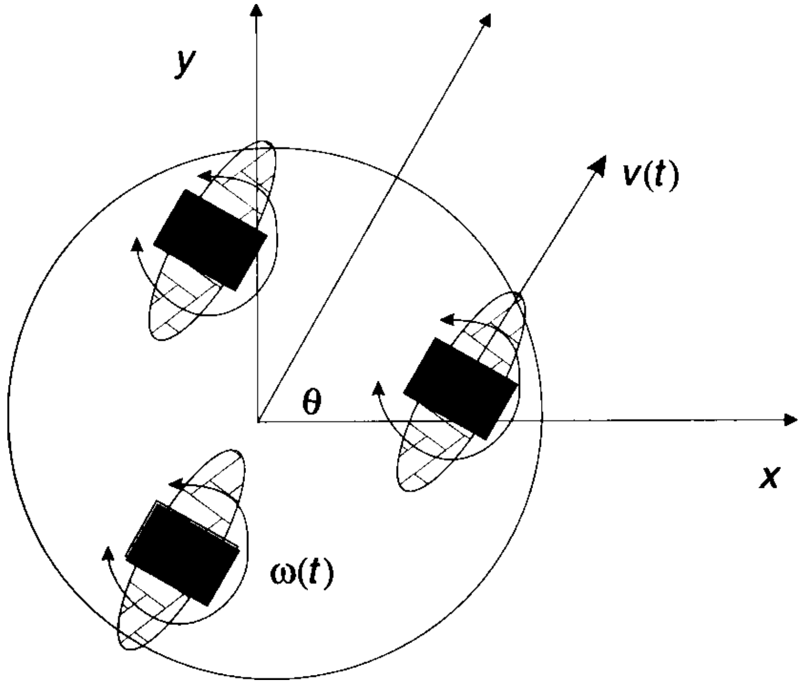
    - **Circular path with ICC (instantaneous center of curvature) on the mid-point between drive wheels**

$$v(t) = 0$$
$$w(t) = \frac{2}{L} v_R(t)$$

Synchronous drive

- In a synchronous drive robot (synchro drive) each wheel is capable of being driven and steered.
- Typical configurations
  - Three steered wheels arranged as vertices of an equilateral triangle often surmounted by a cylindrical platform
  - All the wheels turn and drive in unison
    - This leads to a holonomic behavior
- **Steered wheel**
  - The orientation of the rotation axis can be controlled

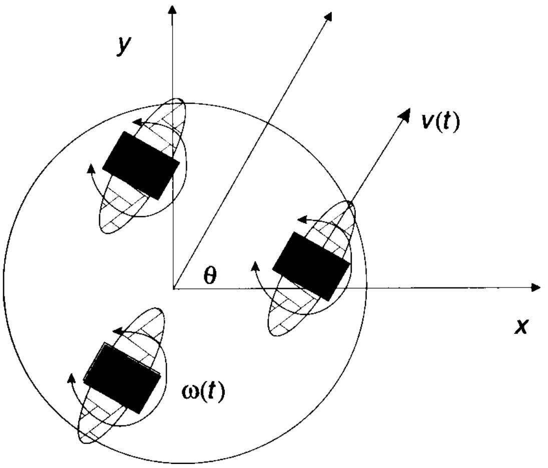




Synchronous drive

- All the wheels turn in unison
- All of the three wheels point in the same direction and turn at the same rate
  - This is typically achieved through the use of a complex collection of belts that physically link the wheels together
- The vehicle controls the direction in which the wheels point and the rate at which they roll
- Because all the wheels remain parallel the synchro drive always rotate about the center of the robot
- The synchro drive robot has the ability to control the orientation θ of their pose diretly.

- Control variables (independent)
  - v(t), w(t)



$$x(t) = \int_0^t v(\sigma)\cos(\theta(\sigma))d\sigma$$

$$y(t) = \int_0^t v(\sigma)\sin(\theta(\sigma))d\sigma$$

$$\theta(t) = \int_0^t w(\sigma)d\sigma$$

- The ICC is always at infinity
- Changing the orientation of the wheels manipulates the direction of ICC

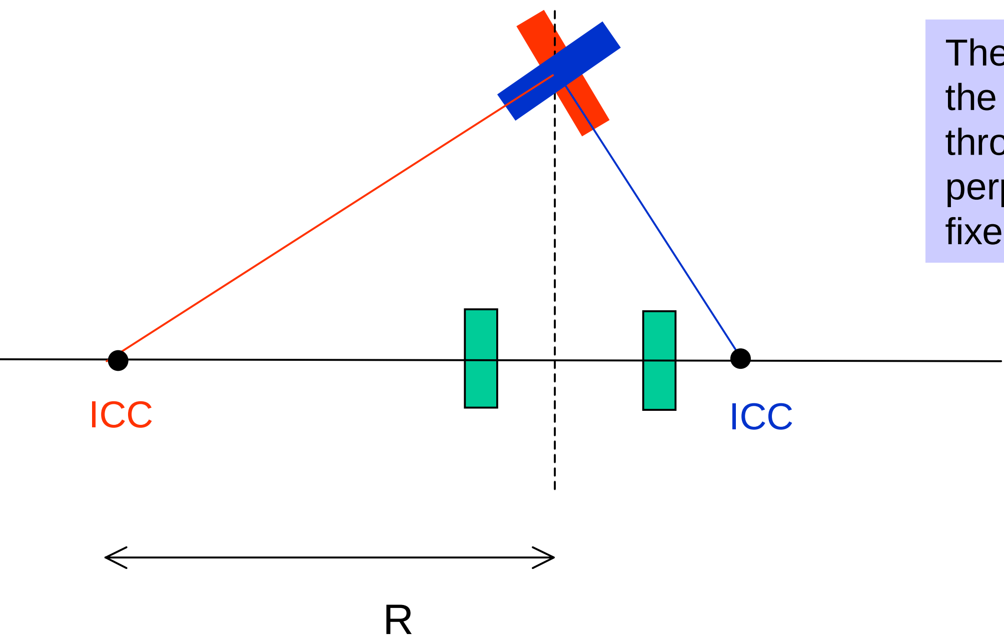
Synchronous Drive

- Particular cases:
  - v(t)=0, w(t)=w=cte. during a time interval Δt
    - **The robot rotates in place by an amount** $w\ \Delta t$
  - v(t)=v, w(t)=0 during a time interval Δt
    - **The robot moves in the direction its pointing a distance** $v\ \Delta t$

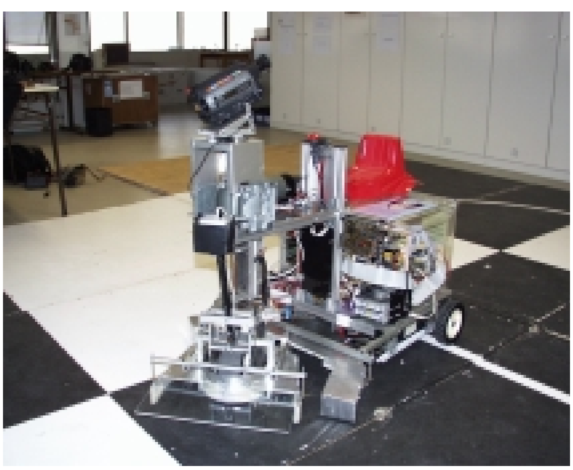
Tricycle

- Three wheels and odometers on the two rear wheels
- Steering and power are provided through the front wheel

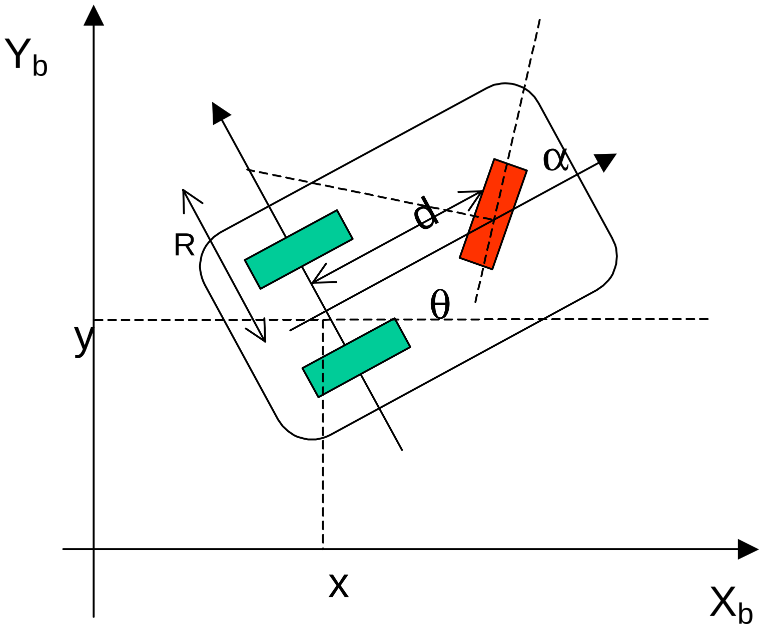
- control variables:
  - steering direction $\alpha(t)$
  - angular velocity of steering wheel $w_s(t)$



The ICC must lie on the line that passes through, and is perpendicular to, the fixed rear wheels



Tricycle



If the steering wheel is set to an angle $\alpha(t)$ from the straight-line direction, the tricycle will rotate with angular velocity w(t) about a point lying a distance R along the line perpendicular to and passing through the rear wheels.

r = steering wheel radius

$$v_s(t) = w_s(t)\, r$$ linear velocity of steering wheel

$$R(t) = d\, \mathrm{tg}\left(\pi/2 - \alpha(t)\right)$$

$$w(t) = \frac{w_s(t)\, r}{\sqrt{d^2 + R(t)^2}}$$

angular velocity of the moving frame relative to the base frame

$$w(t) = \frac{v_s(t)}{d} \sin\alpha(t)$$

Kinematic model in the robot frame

$$\begin{aligned} v_x(t) &= v_s(t)\cos\alpha(t) \\ v_y(t) &= 0 \\ \dot{\theta}(t) &= \frac{v_s(t)}{d}\sin\alpha(t) \end{aligned}$$

→ with no splippage

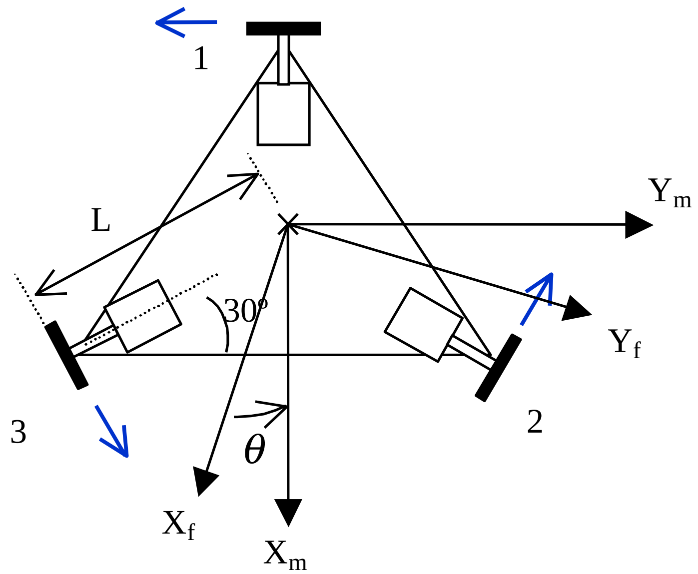
Kinematic model in the world frame

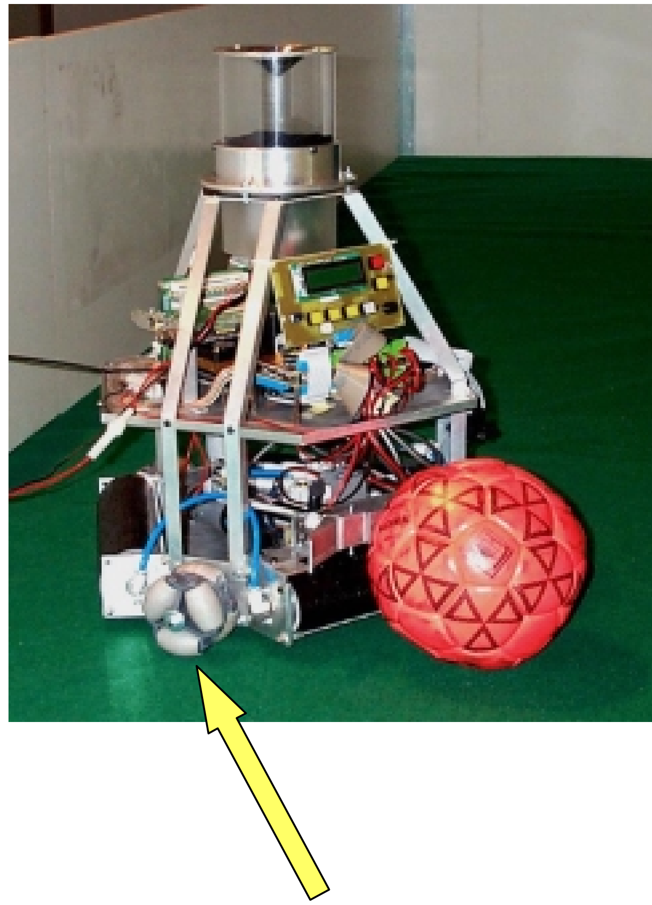
$$\begin{aligned} \dot{x}(t) &= v_s(t)\cos\alpha(t)\cos\theta(t) \\ \dot{y}(t) &= v_s(t)\cos\alpha(t)\sin\theta(t) \\ \dot{\theta}(t) &= \frac{v_s(t)}{d}\sin\alpha(t) \end{aligned}$$

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix}$$

$$\begin{aligned} v(t) &= v_s(t)\cos\alpha(t) \\ w(t) &= \frac{v_s(t)}{d}\sin\alpha(t) \end{aligned}$$

Omnidireccional





Swedish wheel

Kinematic model in the robot frame

$$\begin{bmatrix} V_x \\ V_y \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{\sqrt{3}}r & \frac{1}{\sqrt{3}}r \\ -\frac{2}{3}r & \frac{1}{3}r & \frac{1}{3}r \\ \frac{r}{3L} & \frac{r}{3L} & \frac{r}{3L} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

w_1, w_2, w_3 – angular velocities of the three swedish wheels