

An actigraphy heterogeneous mixture model for sleep assessment.

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Abstract—Wrist actigraphy is a well established procedure to monitor human activity. Among other areas, it has a special relevance in sleep studies where its lightweight and non-intrusive nature make it a valuable tool to access the circadian cycle. While there are several methods to extract information from the data, the differentiation between sleep and wakefulness states is still an open discussion.

In this paper, the characteristics of the movements in the different states are assumed to be intrinsically different. These differences are not simply related with magnitude and movement counting, but due to real differences on the statistical distributions describing the actigraphy data. Thus it is possible to refine the discrimination level when detecting these states.

The proposed methodology to characterize the actigraphy data is based on a mixture of three canonical distributions; i) Exponential, ii) Rayleigh and iii) Gaussian. It is shown that the weights and parameters estimated in each state are organized into almost separable clusters on the feature space. This suggests the ability of the method to discriminate these states based only on the movements recorded on actigraphy data.

I. INTRODUCTION

Normal sleep circadian patterns are fundamental for regular and healthy conditions. The group of pathologies associated with sleep, Sleep disorders, include a wide range of problems, with different origins, symptoms and degrees of impact on health [1].

The diagnosis of these disorders is usually performed with a *polysomnography* (PSG) exam, which involves complex hardware and is highly intrusive to the patient. Other means of diagnosis involves monitorization of the patterns of the patient during his circadian cycle, this monitorization can be done using sleep and dream diaries, portable biosignal recorders and, frequently, actigraphy.

Actigraphy data is obtained with non invasive and highly portable accelerometer sensors, which reflect the motor activity of the subjects. It has become a popular method in sleep studies due to its ability to register behavioral data under normal life conditions and to estimate sleep amounts and sleep continuity in patients with sleep disorders [2].

It has been used with success in the estimation of the shape and characterization of the circadian cycle [3], [4] but its use

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in the estimation of the sleep and wakefulness states is still an open discussion [5].

An extensive literature review on the use of actigraphy for sleep assessment can be found in [6] and [7]. In [8] the ability of actigraphy data for sleep staging is discussed as well as the consistency of the results with PSG.

In this paper we propose a statistical description of the movement based on a mixture of distributions to show that movements during wakefulness and sleep states are intrinsically different.

Purposeless is the key concept of the paper. While movements during sleep state are typically random and without purpose, movements during wakefulness state are coherent and correlated. This empirical observation suggests that movements recorded during different states, apparently similar from temporal and intensity points of view, may present relevant differences from spectral or statistical distribution points of view.

Here, the work from [9], where the description of actigraphy data is tested with several pairs of distributions, is extended. A mixture of distributions is used where the weights and parameters of the components are estimated with an *Expectation Maximization* (EM) based algorithm.

It is shown that these weights and parameters are clustered in different clouds, not completely overlapped, depending on the sleep or wakefulness state.

II. METHODS

Actigraphy data was collected with a Somnowatch device, from Somnomedics, placed at the non-dominant wrist of the subjects with a sampling rate of 1Hz. The core of these devices is a 3D accelerometer that measures the acceleration along 3 orthogonal axis with a configurable output format. Here, the output of the actigraph is the acceleration magnitude.

The actigraphy data used in this study was jointly acquired with PSG data for validation purposes. The hypnogram, obtained from the PSG data by trained technicians, is used as ground truth to identify the sleep and wakefulness states as well the sleep stages in each epoch.

A. Pre-processing

Two pre-processing operations are performed on the data: i) magnitude normalization and ii) activity segmentation.

Magnitude normalization is needed to minimize the inter-patient and intra-patients variability effects. The normalization step is simply a mean subtraction and variance

normalization procedure according with

$$x(n) = \frac{y(n) - \mu_Y}{\sigma_Y} \quad (1)$$

where μ_Y and σ_Y are the mean and standard deviation of the data, respectively.

The second operation, movement segmentation, is performed because the large segments of immobility are useless for activity characterization and sleep staging. They constitute a source of noise and confound factors in the training process of the staging classifier.

A simple threshold based movement detector was implemented to detect movement and extract the corresponding actigraphy data.

Figure 1 displays an example of pre-processed data. Figure 1.a) shows the normalized actigraphy signal and the movement indicator and Figure 1.b) the corresponding hypnogram segment.

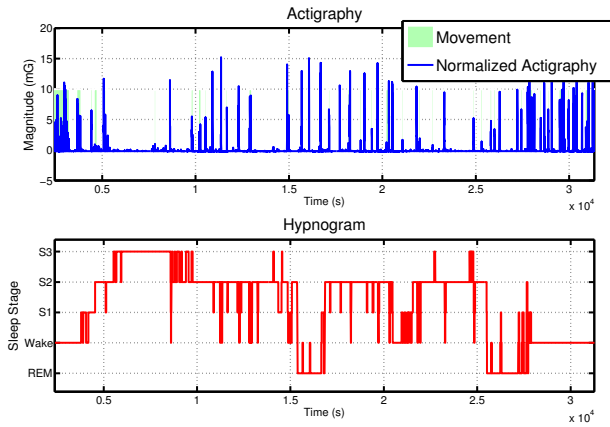


Fig. 1. a) Actigraphy data and detected movements (top) b) and Hypnogram (bottom).

Data acquired from 8 patients was used for analysis. After normalization and movement detection, the segments corresponding to *sleep*, *s*, and *wakefulness*, *w*, states were concatenated into two large arrays respectively.

B. Mixture Distribution

The histogram of movement data, displayed in Figure 2, suggests that the data can be described by a mixture of different probability density functions.

Four combinations of distributions were tested; i) Exponential, Rayleigh and Gaussian (ERG), ii) Exponential and Rayleigh (ER), iii) Exponential and Gaussian (EG) and iv) Rayleigh and Gaussian distribution (RG). The rationale for the selection of four combinations is related with the observation of the histograms of the actigraphy data which present significant changes along the circadian cycle.

Two figures of merit are used to assess the goodness of fit of each combination in order to select the one that better describes the data along the whole cycle.

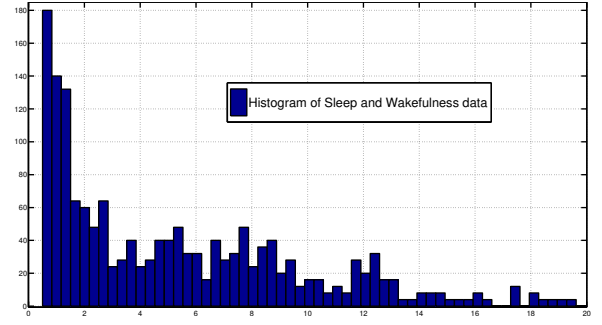


Fig. 2. Histogram of movements during sleep and wake states.

Let $\mathbf{y} = \{y_i\}$, $1 \leq i \leq N$, be a vector with the actigraphy samples for a particular state, sleep or wakefulness, acquired at a constant sampling rate. Each sample, corresponding to the magnitude of the actigraphy data at a given instant, is considered a random variable with the following PDF,

$$p(y_i|\mathbf{W}, \boldsymbol{\theta}) = w_e p(y_i|\lambda) + w_r p(y_i|f) + w_g p(y_i|\mu, \sigma) \quad (2)$$

where $\boldsymbol{\theta} = \{\lambda, f, \mu, \sigma\}$ are the parameters of the components, $\mathbf{W} = \{w_e, w_r, w_g\}$ are the weights of the mixture satisfying the following normalization constraint:

$$w_e + w_r + w_g = 1. \quad (3)$$

$$p_e(y_i|\lambda) = \lambda e^{-\lambda y_i} \quad (4)$$

is an exponential distribution with parameter λ ,

$$p_r(y_i|f) = \frac{y_i}{f} e^{-\frac{y_i^2}{2f}} \quad (5)$$

is a Rayleigh distribution with parameter f and

$$p_g(y_i|\mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y_i - \mu)^2}{2\sigma^2}} \quad (6)$$

is a Gaussian distribution with parameters μ and σ .

The *Maximum Likelihood* (ML) estimation problem of the weights, $\mathbf{W} = \{w_e, w_r, w_g\}$ and parameters, $\boldsymbol{\theta} = \{\lambda, f, \mu, \sigma\}$, assuming statistical independence of the observations is formulated as follows

$$\{\hat{\mathbf{W}}, \hat{\boldsymbol{\theta}}\}^{ML} = \arg \max_{\boldsymbol{\theta}, \mathbf{W}} L(\mathbf{y}|\mathbf{W}, \boldsymbol{\theta}) \quad (7)$$

where

$$L(\mathbf{y}|\mathbf{W}, \boldsymbol{\theta}) = \log(p(\mathbf{y}|\mathbf{W}, \boldsymbol{\theta})) = \log \prod_{i=1}^N p(y_i|\mathbf{W}, \boldsymbol{\theta}) \quad (8)$$

is the likelihood function and

$$p(y_i|\mathbf{W}, \boldsymbol{\theta}) = \sum_j [w_j p_j(y_i|\boldsymbol{\theta}(j))] \quad (9)$$

is the mixture. $\boldsymbol{\theta}(j)$ is the set of parameters associated with the j^{th} component of the mixture.

The maximization of (8) can be efficiently performed using the EM method [10]. The estimation process becomes an iterative process, where new values of $\{\mathbf{W}, \boldsymbol{\theta}\}$ are calculated, on each step, according to:

$$w_j = \frac{1}{N} \sum_{i=1}^N \phi_{i,j} \quad (10)$$

$$\hat{\lambda} = \frac{\sum_{i=1}^N \phi_{i,0}}{\sum_{i=1}^N \phi_{i,0} y_i} \quad (11)$$

$$\hat{f} = \frac{1}{2} \frac{\sum_{i=1}^N y_i^2 \phi_{i,1}}{\sum_{i=1}^N \phi_{i,1}} \quad (12)$$

$$\hat{\mu} = \frac{\sum_{i=1}^N y_i \phi_{i,2}}{\sum_{i=1}^N \phi_{i,2}} \quad (13)$$

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^N \phi_{i,2} (y_i - \hat{\mu})^2}{\sum_{i=1}^N \phi_{i,2}}}$$

where

$$\phi_{i,j} = \frac{w_j^{n-1} p(y_i | \theta^{n-1}(k_j))}{\sum_j w_j^{n-1} p_j(y_i | \theta^{n-1}(j))} \quad (14)$$

The iterative process runs until the fitting error between the estimated mixture distribution and the observed data is lower than a predefined threshold, set experimentally as 10^{-4} .

III. RESULTS

The method presented in the previous section is used to estimate the optimal weights, parameters and combination of distributions to describe actigraphy data, during both sleep and wakefulness states.

The following procedure is repeated for each mixture distribution:

- i) Each of the two (**sleep/wakefulness**) arrays is processed using an overlapping sliding window. The size of the window being 300 samples, corresponding to 5 minute windows.
- ii) On each window, one vector of weights and parameters, $\mathbf{f} = [w_e, w_r, w_g, \lambda, f, \mu, \sigma]$ is computed for each state. By stacking the N vectors, where N is the number of windows, two matrices are obtained, \mathbf{F}^τ , $\tau = \{w, s\}$. Each line \mathbf{f}_n^τ , $0 \leq n \leq N$, corresponds to the vector of weights and parameters computed for the n^{th} window.

Two figures of merit are used to assess the goodness of fit of each mixture distribution; the i) Kullback-Leibler divergence and the ii) normalized cluster distance, given by,

$$n.c.d. = \frac{|\mu_{\mathbf{F}^w} - \mu_{\mathbf{F}^s}|}{\|\Sigma_{\mathbf{F}^w}\|_F + \|\Sigma_{\mathbf{F}^s}\|_F} \quad (15)$$

Where $\mu_{\mathbf{F}^\tau}$ is the mean value calculated over the N lines of \mathbf{F}^τ , $\Sigma_{\mathbf{F}^\tau}$ is the covariance matrix of \mathbf{F}^τ and $\|x\|_F$ is the frobenius norm.

Table I summarizes the obtained results, showing the obtained mean and standard deviation values for the i) kullback-leibler divergence, ii) weights ($\mathbf{w}_0, \mathbf{w}_1, \mathbf{w}_2$) and iii)

Wakefulness				
	ERG	ER	EG	RG
k.l.	1.15 ± 0.31	1.29 ± 0.33	1.13 ± 0.29	1.18 ± 0.29
\bar{w}_0	0.06 ± 0.01	0.6 ± 0.03	0.80 ± 0.03	-
\bar{w}_1	0.58 ± 0.02	0.4 ± 0.03	-	0.61 ± 0.02
\bar{w}_2	0.36 ± 0.01	-	0.20 ± 0.02	0.39 ± 0.01
$\bar{\lambda}$	0.2 ± 0.01	0.32 ± 0.01	0.31 ± 0.01	-
\bar{f}	2.52 ± 0.16	13.58 ± 1.06	-	2.62 ± 0.14
$\bar{\mu}$	6.14 ± 0.13	-	5.50 ± 0.18	6.27 ± 0.10
$\bar{\sigma}$	2.46 ± 0.04	-	2.46 ± 0.06	2.71 ± 0.04
Sleep				
	ERG	ER	EG	RG
k.l.	1.31 ± 0.12	1.54 ± 0.13	1.33 ± 0.12	1.53 ± 0.13
\bar{w}_0	0.22 ± 0.01	0.83 ± 0.03	0.92 ± 0.03	-
\bar{w}_1	0.40 ± 0.01	0.17 ± 0.02	-	0.48 ± 0.02
\bar{w}_2	0.38 ± 0.01	-	0.08 ± 0.02	0.52 ± 0.02
$\bar{\lambda}$	0.18 ± 0.01	0.24 ± 0.01	0.28 ± 0.02	-
\bar{f}	1.25 ± 0.08	1.07 ± 0.12	-	1.48 ± 0.10
$\bar{\mu}$	5.23 ± 0.14	-	6.18 ± 0.31	5.85 ± 0.14
$\bar{\sigma}$	2.07 ± 0.09	-	0.62 ± 0.28	2.94 ± 0.08

TABLE I

SUMMARY OF THE OBTAINED RESULTS.

	ERG	ER	EG	RG
n.c.d.	3.69	2.16	2.33	2.41

TABLE II

CLOUD DISTANCE (15) FOR SLEEP AND WAKEFULNESS STATES.

parameters for the different mixture distributions, for the two different states.

The values obtained for the Kullback-Leibler show that all the mixture distributions are able to properly describe wakefulness data, with EG and ERG mixture distributions yielding the best scores.

Table II shows the obtained results for the normalized cluster distance. While this value is highly dependent on the shape of the cloud of features, assuming a Gaussian distribution, it is useful to give a rough approximation of how separated they are, thus giving an approximate idea of the discriminative power of each mixture. The ERG mixture scored the highest value, followed by RG, EG and finally ER.

The matrix of weights, given by the first 3 columns of \mathbf{F}^τ , obtained for each mixture distribution and for each state is represented in Figure 3. According to (3), the sum of the weights, on each mixture distribution, is 1. Thus Figure 3 only shows 2 weights, in the case of the ERG mixture distribution and 1 weight in the remaining mixture distributions.

The clouds of the weights obtained for the ERG mixture are clearly separated in space, the histograms obtained for the ER, EG and RG mixtures are partially overlapped with RG showing the largest difference between states.

The matrices of parameters, given by the remaining columns of \mathbf{F}^τ , obtained for each mixture and each state are shown in Figure 4.

The clouds of the parameters follow the same pattern

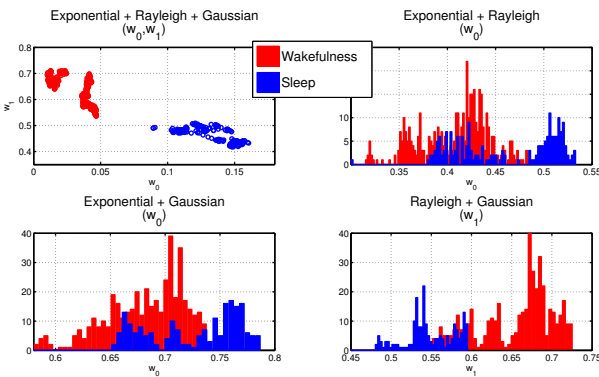


Fig. 3. Clouds of weights for each mixture distribution (left/top) ERG; (right/top) ER; (left/bottom) EG and (right/bottom) RG, and for each state, wakefulness (red) and sleep (blue).

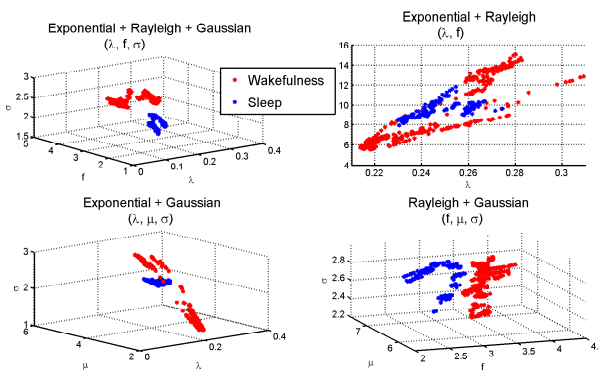


Fig. 4. Clouds of parameters for each mixture distribution, (left/top) ERG; (right/top) ER; (left/bottom) EG and (right/bottom) RG, and for each state, wakefulness (red) and sleep (blue).

observed in the weights. The parameters obtained for the ERG mixture are clearly distinguishable and the remaining mixtures show some overlap with RG mixture showing higher separability.

The previous results graphically confirm the results shown on Table III.

The obtained results suggest that the separability of the weights and parameters of the different mixture distributions can be used with simple discriminative classifiers to discriminate between sleep and wakefulness states. The clustering nature of the clouds of parameters reveals intrinsic differences on the movements recorded in different states thus confirming the initial claim.

Although a special effort was placed in the use of quality data sets a human error factor is always present, specifically in the elaboration of the hypnogram, where classification may sometimes be different between technicians. In this study, the large amount of data that result from each Actigraphy recording was drastically reduced by discarding all the data corresponding to non-movement periods, still the 8 used data sets contained enough data to produce

relevant results.

The obtained results are not clear regarding the best mixture distribution. All the tested mixture distributions were able to fit the data in the different states with ERG and RG yielding the best results regarding the separability of the clouds of weights and parameters.

The obtained results show that it is possible to do a rough estimation of the sleep/wake state based only on the characteristics of the movements recorded on the actigraphy data. While these results alone are not sufficient for a standalone platform, they can be incorporated into existing platforms to improve the accuracy of the classifiers.

IV. CONCLUSIONS

In this work the intrinsic characteristics of the movements recorded on Actigraphy data were characterized with the goal of developing an accurate sleep/wake estimator from Actigraphy and other portable sources of physiological data. It was shown, using the weights and parameters of mixture distributions that the movements during sleep and wakefulness are statistically different and that this difference may be used to discriminate them.

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