Terrain Avoidance Model Predictive Control for Autonomous Rotorcraft

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October 7th, 2008
Outline

1. Introduction
   - Autonomous Rotorcraft
   - Model Predictive Control Approach

2. Model
   - State Space Equation
   - Forces and Moments

3. Controller
   - MPC Control Problem
   - Unconstrained Optimization Problem
   - Algorithm

4. Results
   - Simulation Results
   - Summary
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Introduction

Autonomous Rotorcraft

Applications:
- Low altitude aerial surveillance;
- Automatic infrastructure inspection;
- 3D mapping of unknown environments.

The platform:
- High precision 3D maneuvers;
- Hover and VTOL capabilities;
- Carry multiple sensors.

Challenging Control problem:
- Highly nonlinear and coupled model
- Wide parameter variations over the flight envelope
Introduction
Model Predictive Control Approach

- Helicopter nonlinear model;
- Input and state saturation constraints;
- Terrain avoidance constraint:
  - Virtual repulsive field around vehicle.
- Reformulated into unconstrained problem;
- Solved online at each sampling instant;
- Optimization algorithm:
  - Quasi-Newton search direction;
  - Line search using Wolfe rule.

Helicopter Nonlinear Dynamics
Terrain Avoidance
Input and State Saturation
MPC Controller
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Helicopter Model

Helicopter model:
- 6 DoF Rigid body dynamics;
- Parameterized for the Vario X-treme R/C helicopter

Actuation \( \mathbf{u} = [ \theta_0, \theta_{1c}, \theta_{1s}, \theta_{0t} ]^T \):
- \( \theta_0 \) is the main rotor collective input
- \( \theta_{1c}, \theta_{1s} \) are the main rotor cyclic inputs
- \( \theta_{0t} \) is the tail rotor collective input

State variables:
- \( \mathbf{v} = [ u, v, w ]^T \) – Body-fixed linear velocity
- \( \mathbf{\omega} = [ p, q, r ]^T \) – Body-fixed angular velocity
- \( \mathbf{p} = [ x, y, z ]^T \) – Position
- \( \lambda = [ \phi, \theta, \psi ]^T \) – ZYX Euler angles

State Vector:

\[
\mathbf{x} = \begin{bmatrix}
\mathbf{v} \\
\mathbf{\omega} \\
\mathbf{p} \\
\lambda
\end{bmatrix}
\]
**Helicopter Model**

**State Space Equation**

**State Equation:**

\[
\dot{x} = f_c(x, u) = \begin{bmatrix}
-\omega \times v + \frac{1}{m} [f_h(v, \omega, u) + f_g(\phi, \theta)] \\
-I^{-1}(\omega \times I \omega) + n_h(v, \omega, u) \\
R(\lambda) v \\
Q(\phi, \theta) \omega
\end{bmatrix}
\]

where \( x \in \mathcal{X} \) and \( u \in \mathcal{U} \).
Helicopter Model

Forces and Moments

- Resultant force and moment vectors:

\[
\mathbf{f}_h = \mathbf{f}_{mr} + \mathbf{f}_{tr} + \mathbf{f}_{fus} + \mathbf{f}_{tp} + \mathbf{f}_{fn}
\]

\[
\mathbf{n}_h = \mathbf{n}_{mr} + \mathbf{n}_{tr} + \mathbf{n}_{fus} + \mathbf{n}_{tp} + \mathbf{n}_{fn}
\]

- Main rotor:
  - 1st order pitching dynamics;
  - Steady state flapping dynamics;
  - Lag dynamic neglected;

- Tail rotor pitch, flap and lag dynamics neglected;

- Fuselage modeled as a function of flow velocity, incidence angle and sideslip angle;

- Horizontal tailplane and vertical fin modeled as regular wings.
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MPC Control Problem

- Discrete Model:
  \[ x_{k+1} \approx f(x_k, u_k) = x_k + T_s f_c(x_k, u_k) \]

  
  Prediction Horizon N;

- State Sequence: \( X_k = \{x_k, \ldots , x_{k+N}\} \);

- Input Sequence: \( U_k = \{u_k, \ldots , u_{k+N-1}\} \);

- Reference State Sequence: \( \bar{X}_k = \{\bar{x}_k, \ldots , \bar{x}_{k+N}\} \);

- Reference Input Sequence: \( \bar{U}_k = \{\bar{u}_k, \ldots , \bar{u}_{k+N}\} \);

Where \( X_k, \bar{X}_k \in \mathcal{X}_N \) and \( U_k, \bar{U}_k \in \mathcal{U}_N \) with

\[ \mathcal{X}_N = \{X_k : x_i \in \mathcal{X}, \forall i = k, \ldots , k+N\} \]

\[ \mathcal{U}_N = \{U_k : u_i \in \mathcal{U}, \forall i = k, \ldots , k+N-1\} \]
**MPC Control Problem**

Find, at each iteration $k$, the optimal control sequence $U_k^*$ with horizon $N$, such that the resulting state sequence $X_k^*$ follows the state reference $\bar{X}_k$ without violating the state and input constraints and avoiding collisions, i.e.,

$$U_k^* = \arg \min_{U_k} J_k \quad \text{s.t.} \quad X_k \in \mathbf{X}_N, \ U_k \in \mathbf{U}_N$$

$$F_M(X_k, U_k) = 0, \ F_T(X_k) = 0$$

$$J_k = F_{k+N} + \sum_{i=k}^{k+N-1} L_i, \quad F_i = \frac{1}{2} \tilde{x}_i' P \tilde{x}_i$$

$$L_i = \frac{1}{2} (\tilde{x}_i' Q \tilde{x}_i + \tilde{u}_i' R \tilde{u}_i) \quad \text{with} \quad \tilde{x}_i = x_i - \bar{x}_i \quad \text{and} \quad \tilde{u}_i = u_i - \bar{u}_i$$
Unconstrained Optimization Problem

Saturation Constraint

- Constraint Sets:
  \[ \mathcal{X} = \{ \mathbf{x} \in \mathbb{R}^{n_x} : |x^{(j)}| \leq x^{(j)}_{\text{max}} \ \forall \ j=1,...,n_x \} \]
  \[ \mathcal{U} = \{ \mathbf{u} \in \mathbb{R}^{n_u} : |u^{(l)}| \leq u^{(l)}_{\text{max}} \ \forall \ l=1,...,n_u \} \]

- Penalty function:
  \[ f_S (\mathbf{x}, \mathbf{u}) = \frac{1}{2} \sum_{j=1}^{n_x} h(|x^{(j)}| - x^{(j)}_{\text{max}})^2 w_x^{(j)} + \frac{1}{2} \sum_{l=1}^{n_u} h(|u^{(l)}| - u^{(l)}_{\text{max}})^2 w_u^{(l)} \]

where \( w_x^{(j)}, w_u^{(l)} \in \mathbb{R}^+ \) and \( h(a) = \begin{cases} a, & \text{if } a > 0 \\ 0, & \text{otherwise} \end{cases} \).
Unconstrained Optimization Problem

Terrain Constraint

- Weight minimum distance between helicopter position $\mathbf{p}$ and closest terrain point $\mathbf{p}_m$: $\mathbf{p}_e = \mathbf{p} - \mathbf{p}_m$

- Distance between the terrain and the sphere of radius $r_S$:

  $$g(\mathbf{x}) = \mathbf{p}_e' \mathbf{p}_e - r_S^2$$

- Terrain avoidance constraint function:

  $$f_T(\mathbf{x}) = e^{-g(\mathbf{x})}$$
Saturation and Terrain constraints incorporated in cost functional using penalty methods;

The equivalent optimization problem is given by

\[ U_k^* = \arg \min_{U_k} \bar{J}_k, \quad s.t. \quad F_M(X_k, U_k) = 0 \]

where

\[ \bar{J}_k = \bar{F}_{k+N} + \sum_{i=k}^{k+N-1} \bar{L}_i \]

\[ \bar{F}_i = F_i + f_S(x_i, 0) + f_T(x_i), \quad \bar{L}_i = L_i + f_S(x_i, u_i) + f_T(x_i) \]
Model Constraint solved using Lagrange multipliers $\lambda_i$:

$$H_i = \bar{L}_i + \lambda'_{i+1} f_d(x_i, u_i)$$

$$\bar{J}_k = \bar{F}_{k+N} - \lambda'_{k+N} x_{k+N} + \sum_{i=k+1}^{k+N-1} \left[ H_i - \lambda'_i x_i \right] + H_k$$

Choose:

$$\lambda_{k+N} = \frac{\partial \bar{F}_{k+N}}{\partial x_{k+N}}, \quad \lambda_i = \frac{\partial H_i}{\partial x_i}, \quad \forall i=k+1, \ldots, k+N-1$$

Then, $1^{st}$ order condition of optimality reduced to:

$$\frac{\partial \bar{J}_k}{\partial u_i} = \frac{\partial H_i}{\partial u_i} = 0, \quad \forall i=k, \ldots, k+N-1$$
Algorithm Minimization

Minimization Algorithm

1. Initialize $X_k^{(0)}, \bar{X}_k, U_k^{(0)}$ and $\bar{U}_k$ and set $j = 0$;
2. Compute $\{\lambda_i\}$ and $\left\{ \frac{\partial H_i}{\partial u_i} \right\}$;
3. Compute the search direction $\Delta_k^{(j)}$;
4. Compute the step size $s^{(j)}$ using Wolfe’s rule;
5. Compute $U_k^{(j+1)} = U_k^{(j)} + s^{(j)} \Delta_k^{(j)}$ and $X_k^{(j+1)}$;
6. Test stop condition: if false set $j = j + 1$ and go to step (2); if true apply $u_k^{(j+1)}$ to system.

- Quasi-Newton search direction: $\Delta_k^{(j)} = -D(j) \frac{\partial H_k^{(j)}}{\partial U_k^{(j)}}$. 
Algorithm
Line Search – Wolfe conditions

- Step size optimization subproblem:
  \[ s^* = \arg \min_{s \geq 0} \phi(s) \]

- Cost Functional:
  \[ \phi(s) = \bar{J}_k \left( X_k^{(j+1)}, U_k^{(j+1)} \right), \quad \phi'(s) = \frac{d \phi(s)}{ds} \]

- Wolfe sets (\( \sigma \) and \( \lambda \) are tuning constants):
  \[ \mathcal{A} = \{ s > 0 : \phi(s) \leq \phi(0) + \sigma \phi'(0) s \land \phi'(s) \geq \lambda \phi'(0) \} \]
  \[ \mathcal{L} = \{ s > 0 : \phi(s) > \phi(0) + \sigma \phi'(0) s \} \]
  \[ \mathcal{R} = \{ s > 0 : \phi(s) \leq \phi(0) + \sigma \phi'(0) s \land \phi'(s) < \lambda \phi'(0) \} \]

- Algorithm finds an estimate of \( s^* \) by selecting \( s \in \mathcal{A} \).
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Simulation Results

Implementation

- Horizon $N = 70$;
- Sample time $T_s = 0.02\, s$;
- Simulation trajectory:
  - Hover at initial position;
  - Straight line until final position;
  - Hover at final position;
- Terrain simulates a river bed;
  - Reference trajectory collides twice with terrain;
  - Simplified nonlinear model used in MPC;
  - Full nonlinear model used as the plant.
Simulation Results

Position

Actuation

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Simulation Results

Helicopter Trajectory

Simulating: t = 000.00/110.00 seg.
Summary

- The results indicate that the presented methodology can achieve effective terrain avoidance while steering the vehicle along a reference trajectory
  - Full nonlinear model of the helicopter;
  - Repulsive field constraint for terrain avoidance;
  - Input and State saturation constraints;
  - Quasi-Newton Algorithm for minimization;
  - Line search algorithm reduces computational effort.

- Further research:
  - Nonlinear model simplification;
  - Efficient computation of the closest terrain point;
  - Tuning sampling frequency and prediction horizon of the MPC controller.
The end

- Thank you for your time.
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